RAND Journal of Economics
Vol. 31, No. 1, Spring 2000
pp. 22–42

Stock-related compensation and product-market competition

Giancarlo Spagnolo*

I show that as long as the stock market has perfect foresight, profits are distributed as dividends, and incentives are paid more than once or are deferred, stock-related compensation packages are strong incentives for managers to support tacit collusive agreements in repeated oligopolies. The stock market anticipates the losses from punishment phases and discounts them on stock prices, reducing managers' short-run gains from any deviation. When deferred, stock-related incentives may remove all managers' short-run gains from deviation, making collusion supportable at any discount factor. The results hold with managerial contracts of any length.

1. Introduction

■ In a highly discussed empirical study, Jensen and Murphy (1990) showed that until the end of the 1980s, and contrary to the predictions of early agency theory, U.S. top managers' compensation had on average a very low pay-performance sensitivity. Kaplan (1994a, 1994b) found analogous results for other developed countries, such as Germany and Japan. These surprising findings led to concerns about the welfare implications of most common governance practices, since low-powered managerial incentives tend to soften product-market competition (Aggarwal and Samwick, 1999; Spagnolo, 1996).

More recently, Hall and Liebman (1998) have shown that the pay-performance sensitivity of U.S. top managers' compensation has increased substantially in the last decade, mainly because of a widespread adoption of stock-related incentives, such as stock option plans. Stock-based managerial incentives are believed to be a powerful tool by which owners can motivate managers to work hard, to take risks, and to take into account the long-run effects of their choices on firms' profitability (see, for example, Bhagat, Brickley, and Lease, 1985; Murphy, 1985; Jackson and Lazear, 1991; and Scholes, 1991; and Holmström and Tirole, 1993). What about their effects on

* Stockholm School of Economics; Giancarlo.Spagnolo@hhs.se.

A previous version of this article circulated under the title ''Shareholder-Value Maximization and Tacit Collusion.'' I am grateful to Jörgen W. Weibull and Tore Ellingsen for invaluable guidance, and to Armando Gomes, Bengt Holmström, Peter Högfeldt, Yeongjae Kang, Klaus Schmidt, the Editor Jennifer Reinganum, and two anonymous referees for helpful comments. Thanks also to participants at the 1997 Stockholm IO Workshop, the 1998 EFA meeting in Fontainebleau, the 1998 EARIE meeting in Copenhagen, and at seminars at the Stockholm School of Economics and the University of Munich. All errors are mine. Financial support from the Swedish Competition Authority and the European Commission (Marie Curie Fellowship) is gratefully acknowledged.

product markets? Does this trend toward stock-based incentives imply a more competitive attitude on the part of managers, so that concerns about tacit collusion and social welfare can be abandoned, at least in the United States?

The results of this article suggest that this may not be the case. My model shows that as long as agents in financial markets have rational expectations and firms pay out profits as dividends, most common stock-based managerial compensation plans greatly facilitate tacit collusion in long-run oligopolies. I find that stock-related compensation reduces managers' incentives to break any tacit agreement in any repeated oligopoly, and it may make the joint monopoly agreement supportable at any level of the discount factor.

The phenomenon of tacit collusion in long-run oligopolies has been fruitfully studied in the last three decades within a discounted repeated-games framework (classical references include Friedman, 1971; Aumann and Shapley, 1992; Rubinstein, 1979; Green and Porter, 1984; Fudenberg and Maskin, 1986; Rotemberg and Saloner, 1986; and Abreu, 1986, 1988). However, most supergame-theoretic analyses of collusion confine themselves to the standard assumption that firms maximize the discounted sum of expected per-period profits. In the real world, many interacting factors affect firms' objective function, and consequently their ability to collude. Among these factors, the most important are probably managerial incentives.[1]

A number of authors have already explored the strategic effects of delegating decision power to managers with preferences/incentives different from those of owners in oligopolies (for example, Vickers, 1985; Fershtman, 1985; Fershtman and Judd, 1987; Sklivas, 1987; Fershtman, Judd, and Kalai, 1991; Katz, 1991; and Reitman, 1993). Most contributions to this literature focus on the strategic effects of managerial incentives in two-stage models, where owners simultaneously choose their managers' incentive schemes before a one-shot oligopolistic market interaction between manager-led firms. In Fershtman and Judd (1987) and Sklivas (1987), firm owners can precommit to a more aggressive market behavior by choosing the parameters of a managerial contract that is linear in profits and sales revenue. In the case of quantity competition, the simultaneous attempts to gain a strategic advantage by precommitting through managerial incentives offset one another and lead to higher output and lower profits than in the standard Cournot-Nash equilibrium.

In probably the closest article in spirit to the present one, Reitman (1993) has shown that if one lets owners introduce stock options in a Fershtman-Judd–Sklivas type model, this may curb managers' overly aggressive behavior and bring back the original Cournot-Nash equilibrium. This result is due to the nonlinearity of stock options in stock price, which induces a discontinuity in managers' best-response function and generates equilibria other than the Fershtman-Judd–Sklivas one. The Pareto-dominant among the symmetric equilibria corresponds to the no-delegation Cournot-Nash equilibrium, so that if managers coordinate on this equilibrium, the ''delegation prisoner's dilemma'' identified by Fershtman-Judd and Sklivas disappears and, eventually, the ability to precommit through managerial incentives does not affect the outcome of the Cournot game.

I depart from most previous work on delegation by following Spagnolo (1996) in allowing for repeated interaction, so that tacit collusion can be analyzed with the tools of repeated games.

[1] This was recognized early by scholars interested in firm behavior, and it led to the so-called managerial theory of the firm (Simon, 1957; Baumol, 1958; Cyert and March, 1963; Marris, 1964; Williamson, 1964; and Jensen and Meckling, 1976).

I focus on stock-based compensation plans as commonly designed in the real world according to Kole's (1997) empirical findings. These plans are typically quite liquid (when not in cash, they have few restrictions on resale or transfers of shares) and pay managers stock-based bonuses for several consecutive years.

The pro-collusive effect that I identify is linked to the fact—forcefully stressed by Holmström and Tirole (1993)—that the stock price incorporates additional information with respect to a firm's profits, information strictly related to the firm's future profitability. Incentive schemes based on stock price link managers' *present* compensation to the stock market's expectations about firms' *future* profitability. When a breach of a tacit collusive agreement occurs, a stock market with rational expectations anticipates the negative effect of the breach on firms' future profitability linked to the forthcoming price/quantity war, and immediately discounts it on stock prices (for a real-world example see Laing (1997)). Because this effect occurs in the very same period in which a manager deviates from a collusive agreement, incentives linked to stock price directly reduce managers' short-run gains from deviation.

Furthermore, when stock-based incentives are deferred, as they often are in reality, the first pro-collusive effect is reinforced by the fact that the already limited beneficial effect on the stock price of short-run profits from a unilateral deviation may be completely gone at the time when the manager receives the bonus. Then, the manager is left with no incentive whatsoever to deviate, which further stabilizes collusive agreements.

Interestingly, we also find that these pro-collusive effects are not reduced, and may well be reinforced, when managerial contracts are short term.

Although this article is close in spirit to Reitman (1993), the effects identified here are very different from the effect discussed there. Both the ''expectations effect'' and the ''deferred incentives effect'' are not linked to the nonlinearity of stock options; they apply to any form of managerial compensation that is positively related to stock price. Also, the effects of managerial incentives discussed here have a strong impact on the equilibrium outcome of the oligopoly game. In my model, deferred stock-related managerial incentives make the joint monopoly outcome supportable even when owners or profit-maximizing managers could not support any collusive agreement. Moreover, the results in this article are not specific to Cournot competition; they extend to other kinds of repeated oligopoly.

I follow the literature on strategic delegation by assuming observable and binding managerial incentives. This assumption has been criticized on the ground of its robustness with respect to secret renegotiation (Dewatripont, 1988; Katz, 1991). However, as Reitman (1993) also made clear, for the case that I am focusing on, this assumption is close to reality. The adoption of stock-based managerial incentive plans, such as stock options, normally requires shareholders' approval. Shareholders' approval must be obtained in open shareholders' meetings, and these make stock-based incentives and their renegotiation almost public information.[2]

Finally, the results of this article are related to but quite distinct from those in Fershtman, Judd, and Kalai (1991), Polo and Tedeschi (1992), and Aggarwal and Samwick (1999). Fershtman, Judd, and Kalai obtain a full ''folk theorem'' for two-stage observable delegation games by using ''target compensation functions'' that award agents a fixed prize as long as managers keep their principals' utility above a certain

[2] Consider, for example, the recent worldwide discussions on the stock-option plan in Walt Disney management's new compensation package. The results of this article would be of interest even if secret renegotiation were possible, as the *costs* typically linked to contract renegotiation would still give commitment value to managerial incentives (see the discussion in Section 6).

level. Polo and Tedeschi and Aggarwal and Samwick obtain cooperative outcomes in two-stage delegation games by allowing managerial contracts to be positively related to competing firms' profits. Here I work with repeated oligopoly models instead, and I obtain full collusion at any discount factor with the empirically observed stock-related managerial incentive plans, which are not target compensation functions and are conditional only on the firm's own stock price.

The rest of the article is organized as follows: Section 2 presents the model, Section 3 discusses the pro-collusive effect linked to stock market expectations, Section 4 considers deferred stock-based incentives, Section 5 endogenizes owners' choice of managers' contracts and discusses the effect of their length, Section 6 extends and discusses the results, and Section 7 briefly concludes. All proofs are in the Appendix.

2. The model

■ **Product market.** There are N symmetric firms, indexed by the subscript i. Market structure is a standard Cournot oligopoly (stage game) infinitely repeated in discrete time under complete and perfect information.

Let $\pi_i(q_i, q_{-i}) = P(q_i + q_{-i})q_i - c(q_i)$ denote firm i's static (stage-game's) profit function, where q_i represents firm i's output, q_{-i} the quantity produced by the other $N - 1$ firms, $P(\cdot)$ the inverse demand function, and $c(\cdot)$ firms' cost function.

I assume that the inverse demand function satisfies $P' < 0$ and $P'' \geq 0$, that profits are concave in firms' own output, and that marginal profits are decreasing in rivals' output, so that static reaction functions are continuous and downward sloping.

Let $\pi_i^N = \pi_i(q_i^N, q_{-i}^N)$ denote firm i's static (stage-game's) profits when firms produce the Cournot-Nash equilibrium output vector $q^N = (q_1^N, \ldots, q_n^N)$, $\pi_i^A = \pi_i(q_i^A, q_{-i}^A)$ denote owner i's static payoff from a stationary tacit agreement A to restrict production to the vector $q^A = (q_1^A, \ldots, q_n^A)$, and $\hat{\pi}_i^A = \pi_i(\hat{q}_i(q_{-i}^A), q_{-i}^A)$ denote his static payoffs from unilaterally deviating from A by producing the static best response output $\hat{q}_i(q_{-i}^A)$. Analogously, $\pi_i^M = \pi_i(q_i^M, q_{-i}^M)$ will denote firm i's profits at the joint monopoly market outcome q^M, and $\hat{\pi}_i^M = \pi_i(\hat{q}_i(q_{-i}^M), q_{-i}^M)$ will denote static payoffs from unilaterally deviating from the joint monopoly collusive agreement.

Time is indexed by the superscript $t = 1, 2, 3 \ldots$ (the time superscript is absent when I refer to a representative period), and δ denotes the intertemporal discount factor common to all agents, owners and managers. I assume that at each point in time t each agent maximizes the discounted sum of expected monetary gains. So each owner i maximizes the discounted sum of firm i's expected profits $U_i^t = \sum_{\tau=1}^{\infty} \delta^\tau \pi_i^{t+\tau}$.

To simplify exposition I focus on stationary collusive agreements enforced by "unrelenting" trigger strategies, that is, by the threat of reverting to the noncooperative Cournot-Nash equilibrium forever (Friedman, 1971).[3] Also, to make things more interesting, I assume throughout that the discount factor is too low for owners, or for managers with incentives in line with owners' objectives ("profit-maximizing managers" from now on), to support the joint monopoly collusive agreement in subgame-perfect equilibrium.

□ **Financial market.** I assume the following:

1. The stock market is perfectly informed, rational, and skilled in game theory: it fully understands equilibria selected in the product market.

[3] In Section 6 and in Appendix 2 of Spagnolo (1998a) I show that the choice of more sophisticated strategies (e.g., finite length, "optimal," or renegotiation-proof punishment strategies) does not affect my conclusions. Also, at the cost of a more cumbersome exposition the results can easily be extended to encompass nonstationary collusive agreements.

2. The value of a firm (of its shares) in one period depends positively upon the discounted profit stream it is expected to generate and on the realized profits that have not yet been distributed as dividends (I assume no physical assets, to simplify exposition).

3. At the end of each period, realized profits are paid out to shareholders as dividends (but see Section 5).

Under these assumptions the price of one share of firm i, P_i^t, at the end of period t before period t's dividends are paid out is

$$P_i^t = \frac{1}{\varphi_i}\left[\pi_i^t + \sum_{\tau=1}^{\infty} \delta^\tau \pi_i^{t+\tau}\right],$$

where φ_i is the (large) number of firm i's shares.

□ **Managerial contracts.** According to Kole (1997), most common stock-based managerial incentive plans are relatively liquid, such as stock options with stock appreciation rights (SARs) or share-performance cash bonuses. In most cases the effect of these incentive plans is deferred and distributed in time, probably to reduce the much advertised risk of costly managerial ‘‘short-termism’’ in investment choices (see, for example, Narayanan, 1985; Stein, 1989; Bebchuck and Stole, 1993; and Bizjak, Brickley, and Coles, 1993). For example, for stock options the typical vesting schedule includes a ‘‘wait to exercise’’ of 12 months for the first quarter of the award, after which the remainder of the award becomes available in equal installments over the next three years. I shall focus mainly on the product-market effects of these more common stock-related incentive plans.[4]

I assume the following:

1. Firms are run by managers with observable incentive contracts that may be linked to stock price.[5]

2. When managers receive compensation in stock or stock options, they are not required to keep the firms' shares; the instant in which they receive their compensation, they sell their shares or options in order to diversify their portfolio.

3. When a manager is indifferent about available actions because they all lead to the same wage, the manager chooses the action that maximizes the owner's objective function.

□ **Useful benchmarks.** Given the common discount factor, any collusive agreement A is sustainable in subgame-perfect equilibrium by profit-maximizing managers as long as discounted expected profits from sticking to the agreement exceed expected profits from deviating, that is,

$$\frac{\pi_i^A}{1-\delta} \geq \hat{\pi}_i^A + \frac{\delta \pi_i^N}{1-\delta}. \tag{1}$$

Let $\overline{A}$ denote the ‘‘most collusive’’ symmetric agreement that profit-maximizing managers can support at the given discount factor, the one that makes (1) hold as an

[4] Restricted stock awards and stock option plans with restrictions on resale/transfer of the shares are used by a minority of firms. I discuss the product-market effects of these other incentives in Section 6 (managerial ownership).

[5] That is, in this version of the article I follow the literature by neglecting the possibility that an owner does not hire a manager (Fershtman and Judd, 1987; Sklivas, 1987; Reitman, 1993). See Spagnolo (1998a) for an alternative approach.

equality, where $\pi_i^{\overline{A}} < \pi_i^M$. Alternatively, one can rewrite (1) in terms of the minimum level of the discount factor $\underline{\delta}^A$ at which profit-maximizing managers can support a given agreement A, that is,

$$\delta \geq \underline{\delta}^A = \frac{\hat{\pi}_i^A - \pi_i^A}{\hat{\pi}_i^A - \pi_i^N}.$$

It is useful to state a simple lemma.

Lemma 1. The Cournot-Nash equilibrium outcome (the equilibrium outcome of the stage game played by profit-maximizing managers) is also a Nash equilibrium outcome of the stage game played by managers under incentive contracts positively related to stock price.

The statement follows directly from the assumptions (a simple proof is in Spagnolo, 1998a). In the static interaction, if all other managers choose the Cournot-Nash production level, then a manager paid as a function of stock price cannot gain by choosing a different production level: any other choice will reduce the firm's profits, the stock price, and therefore the manager's compensation. This lemma makes sure that the reversion to the static Cournot-Nash equilibrium remains a credible punishment strategy when managers under stock-based compensation are running the firms, and it allows me to study the effects of these incentives on firms' ability to collude by plugging managers' compensation function into condition (1).

3. Stock-related compensation, expectations, and collusion

■ In this and in the next section I analyze the product-market effects of stock-related managerial compensation packages by taking them as given, as observed in reality. The results of these sections are the crucial ones, since they unveil an important side effect of empirically observed governance practices. In Section 5 I endogenize managerial contracts by leaving owners free to choose managerial incentives other than stock-related ones, and show that pro-collusive stock-related incentives are the outcome of a collusive equilibrium.[6]

□ **''Small'' compensation packages.** I first follow the literature (e.g., Fershtman and Judd, 1987; Sklivas, 1987; Reitman, 1993) by neglecting the direct effect of managers' compensation on firms' profits and stock prices. In my model, as in most previous models of strategic delegation, the managers' task is only to set the level of a strategic variable. Absent issues of managerial effort and moral hazard, there is no reason for owners to give managers large amounts of incentive pay, since as long as the manager is paid in the right way at the margin, a small incentive component with negligible effects on profits and stock prices is sufficient to induce the desired behavior.[7]

Consider the following class of stock-related managerial incentive contracts.

---

[6] Though I could not seriously argue that stock-related incentives are popular because of their pro-collusive effect. I believe that few practitioners, if any, are aware of it. Independent of why stock-related incentives are adopted in reality, it is important to know that a socially harmful side effect may contribute to the improvements in profitability they appear to generate.

[7] In what follows I will focus only on the incentive part of managers' compensation $f(P)$. However, managers' actual compensation will be some function $A + Bf(P)$, where the parameters A and B can be freely set to reflect conditions on the managers' labor market. Whatever A and B are, and therefore however small B is, managerial behavior will still be driven by only the marginal incentive component $f(P)$ (see Fershtman and Judd (1987)).

Definition 1. Incentive contract class A (ICA): In each period t, the manager of firm i receives a compensation positively related to the stock price $f_i(P_i^t)$—where f_i is *any* monotone and strictly increasing function—*before* period-t profits are paid out as dividends.

Disregarding the negligible effects of small managerial bonuses on firms' stock prices, and given that under ICA-type contracts managers are paid before the distribution of dividends, the value of a firm's share when they receive their compensation is exactly as in the example in Section 2. Then, the incentive-compatibility condition for a stationary collusive agreement A to be supportable by the manager of firm i under a compensation package in the ICA class is

$$\frac{1}{1-\delta} f_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\delta}\right)\right] > f_i\left[\frac{1}{\varphi_i}\left(\hat{\pi}_i^A + \frac{\delta\pi_i^N}{1-\delta}\right)\right] + \frac{\delta}{1-\delta} f_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^N}{1-\delta}\right)\right], \tag{2}$$

where the inequality is strict because of the third assumption in the ''managerial contracts'' subsection of Section 2.

I can now state the first result.

Proposition 1. Suppose firms are led by managers under incentive contracts in the class ICA and managerial compensation has a negligible effect on stock price. Then

(i) the minimum discount factor at which any collusive agreement can be supported in subgame-perfect equilibrium is strictly lower than when firms are led by profit-maximizing managers; and

(ii) for a given discount factor, more profitable collusive agreements become supportable than when firms are led by profit-maximizing managers.

The intuition behind this result is the following. The short-run incentive to deviate from any collusive agreement is lower for a manager under an ICA-type contract than for a profit-maximizing one because the value of the shares of a firm that deviates from a collusive agreement does not increase as much as short-run profits in the period in which the deviation occurs. This is because, as noted by Holmström and Tirole (1993), the stock price contains more information than accounting profits, and in the case of a deviation the additional information is about the forthcoming punishment phase, that is, bad news. The stock market forecasts that the deviation will be followed by a production war leading to a period of low profits, and it adjusts firms' stock prices accordingly. Therefore, a negative effect of the punishment phase occurs on (deviating and nondeviating) managers' compensation already in the *same* period in which the deviation occurs. In addition, expected stock price and related bonuses in the periods that follow the deviation are low because gains from deviation are distributed and per-period profits are depressed by the punishment phase. These effects make managers under stock-related compensation more prone to collude than profit-maximizing ones.

□ **''Large'' compensation packages.** I wrote before that I believe stock-related managerial incentives are adopted in reality not because they may be pro-collusive, but because of their positive effects on managerial behavior already highlighted in the literature (see the Introduction). To realize these other effects, however, the amount of stock-related pay may matter. Indeed, in some real-world cases, managerial compensation packages appear ''heavy'' enough to noticeably influence the firms' stock price. To take these cases into account, I turn now to stock-related incentives which, for unmodelled reasons, are large enough to have nonnegligible direct effects on the firm's stock price.

In this case, when the manager is hired under a contract in the class ICA, the price of one share of firm i, P_i^t, at the end of period t before period t's dividends are paid out is

$$P_i^t = \frac{1}{\varphi_i}\left[\pi_i^t - f(P_i^t) + \sum_{\tau=1}^{\infty} \delta^\tau(\pi_i^{t+\tau} - f(P_i^{t+\tau}))\right]. \tag{3}$$

To compare the condition for a manager under stock-related compensation to be willing to sustain collusion with the benchmark condition (1), I need to calculate the stock price, and because of the recursive structure of (3) I need to be more precise about the function f. The most natural thing to do, in line with previous work on delegation, is to let f be a linear function.

Definition 2. Incentive contract class A^L (ICA^L): In each period t, the manager of firm i receives $\alpha_i P_i^t$, where $0 < \alpha_i < \varphi_i$, *before* period-t profits are paid out as dividends.

The incentive-compatibility condition for a stationary collusive agreement A to be supportable by the manager of firm i under a compensation package of the ICA^L type is

$$\frac{1}{1-\delta}\alpha_i P_i^A > \alpha_i \hat{P}_i^A + \frac{\delta}{1-\delta}\alpha_i P_i^N, \tag{4}$$

where

$$P_i^A = \frac{1}{\varphi_i}\left(\frac{\pi_i^A - \dfrac{\alpha_i}{\varphi_i}P_i^A}{1-\delta}\right), \qquad \Rightarrow P_i^A = \frac{1}{\varphi_i}\frac{K\pi_i^A}{1-\delta},$$

with

$$K = \frac{\varphi_i(1-\delta)}{\varphi_i(1-\delta) + \dfrac{\alpha_i}{\varphi_i}} < 1;$$

$$P_i^N = \frac{1}{\varphi_i}\left(\frac{\pi_i^N - \dfrac{\alpha_i}{\varphi_i}P_i^N}{1-\delta}\right), \qquad \Rightarrow P_i^N = \frac{1}{\varphi_i}\frac{K\pi_i^N}{1-\delta}; \quad \text{and}$$

$$\hat{P}_i^A = \frac{1}{\varphi_i}\left(\hat{\pi}_i^A - \frac{\alpha_i}{\varphi_i}\hat{P}_i^A + \delta\frac{\pi_i^N - \dfrac{\alpha_i}{\varphi_i}P_i^N}{1-\delta}\right), \qquad \Rightarrow \hat{P}_i^A = \frac{1}{\varphi_i}\frac{\hat{\pi}_i^A + \delta\dfrac{K\pi_i^N}{1-\delta}}{1 + \dfrac{1}{\varphi_i}\dfrac{\alpha_i}{\varphi_i}}.$$

I can now state the following result.

Proposition 2. Suppose firms are led by managers under incentive contracts in the class ICA^L. Then

(i) the minimum discount factor at which any collusive agreement can be supported in subgame-perfect equilibrium is strictly lower than when firms are led by profit-maximizing managers; and

(ii) for a given discount factor, more profitable collusive agreements become supportable than when firms are led by profit-maximizing managers.

Although the proof is algebraically more cumbersome, the intuition behind the result is exactly as for Proposition 1. The case of linear stock-related incentives with nonnegligible effects on the stock price does not differ in any substantive way from that of stock-related incentives with negligible effects on the stock price.

For the sake of crispness, in the remainder of the article I shall follow the literature in focusing on ''small'' compensation packages. Corresponding results for the case of ''large'' compensation packages can be obtained as done in this section (and in the proof of Proposition 2 in the Appendix).

□ **Stock options.** To make the results more concrete, consider the case of the most popular stock-based managerial incentives: stock option plans. Stock options are not strictly increasing functions of the stock price (for all strike prices above the stock price, the value of the option is constant and equal to zero), therefore I could not simply apply the results in the previous subsections to this case.

Definition 3. Incentive contract class $A^{\underline{P}}$ ($ICA^{\underline{P}}$): In each period t, the manager receives the right to buy a number γ_i of shares at a predetermined price $\underline{P}_i$, both of which are constant across time periods, *before* period-t profits are paid out as dividends.

Again, because under these contracts managers get paid before the distribution of dividends, the value of a share when they can cash their stock options includes that period's profits. Then the incentive-compatibility condition for a stationary collusive agreement A to be supportable by the manager of firm i under a compensation package of the $ICA^{\underline{P}}$ type is

$$\frac{1}{1-\delta}\max\left\{\gamma_i\left[\frac{1}{\varphi_i}\frac{\pi_i^A}{1-\delta}-\underline{P}_i\right],0\right\} > \max\left\{\gamma_i\left[\frac{1}{\varphi_i}\left(\hat{\pi}_i^A+\frac{\delta\pi_i^N}{1-\delta}\right)-\underline{P}_i\right],0\right\} + \frac{\delta}{1-\delta}\max\left\{\gamma_i\left[\frac{1}{\varphi_i}\frac{\pi_i^N}{1-\delta}-\underline{P}_i\right],0\right\}. \tag{5}$$

In each period the stock options are ''in the money'' (valuable) if the price of the shares P_i^t at the end of the period is higher than $\underline{P}_i$. I can state the following result.

Proposition 3. Suppose firms are led by managers under incentive contracts in the class $ICA^{\underline{P}}$, with $\underline{P}_i < (\pi_i^A/(1-\delta))/\varphi_i \; \forall i$. Then

(i) the minimum discount factor at which any collusive agreement delivering per-period profits π_i^A can be supported in subgame-perfect equilibrium is strictly lower than when firms are led by profit-maximizing managers; and

(ii) for a given discount factor, more profitable collusive agreements become supportable than when firms are led by profit-maximizing managers.

Again, the intuition is as for Proposition 1. One can also state the following corollary.

Corollary 1. Suppose the repeated oligopoly game is played by managers under incentive contracts in the class $ICA^{\underline{P}}$. Then, the minimum discount factor at which any collusive agreement can be supported in subgame-perfect equilibrium is independent of γ_i and is minimized when $\underline{P}_i \leq (\pi_i^N/(1-\delta))/\varphi_i \; \forall i$.

(As for previous results, a corresponding statement holds for the most profitable agreement supportable at a given discount factor.) The corollary implies that the pro-collusive effect is stronger when the strike price is so generous that stock options are valuable whatever collusive equilibrium is chosen.

4. Deferred stock-related compensation

■ **The general case.** Consider a slightly different class of contracts by which in each period managers receive their stock-related bonuses only after having distributed that period's profits as dividends.

Definition 4. Incentive contract class B (ICB): In each period t, the manager receives a compensation positively related to stock price $f_i(P_i^t)$—where f_i is any monotone and strictly increasing function—*after* period-t profits are paid out as dividends.

Because managers get paid after the distribution of dividends, the value of the shares when they cash their options does not incorporate present profits π_i^t. So when managers under ICB-type contracts cash their bonuses, the stock price is

$$P_i^t = \frac{1}{\varphi_i}\left[\sum_{\tau=1}^{\infty} \delta^\tau \pi_i^{t+\tau}\right].$$

Also, consider stock-related incentive plans such that managers can cash the bonuses only some time after having left the firm. This form of compensation is often introduced to keep managers (who may be planning to leave the firm) from taking actions against the long-run interest of shareholders in order to improve their short-run market valuation, and to maintain an incentive for managers close to retirement to work hard. To keep things simple I assume that managers face a constant per-period probability $(1 - \eta)$ of leaving the firm (because of a take-over, say, or because they find a better job).

Definition 5. Incentive contract class C (ICC): In each period, the manager receives a wage, which I normalize to zero, and in the period after he stops working for the firm, say τ periods after he started, he receives additional compensation positively related to stock price $(1 + r)_i^\tau f_i(P_i^\tau)$—where f_i is *any* monotone increasing function and $1/1 + r = \delta$.

Then I can state the following result.

Proposition 4. Suppose firms are led by managers under incentive contracts in the class ICB or ICC. Then the joint monopoly collusive agreement can be supported in subgame-perfect equilibrium at any level of the discount factor.

The intuition behind the proposition is somewhat analogous to that behind Propositions 1 to 3, but here the mechanism is more extreme. For a manager under a contract in the class ICB or ICC, there is no incentive whatsoever to deviate from collusion. After short-run profits from a deviation are paid out as dividends, the price of the shares of the deviating firm (and therefore its manager's compensation) depends *only* on stock market expectations about the firm's future profitability, and therefore it falls. Managers under contracts in the classes ICB and ICC incur a net loss when they deviate from a collusive agreement, without ever being able to capture any of the short-run gains from deviating.[8]

[8] Note that nothing changes if managers under ICC-type contracts know exactly when they will stop working for their firms. What is important is that after managers have left, firms go on producing so that stock market expectations about firms' future profitability can influence the leaving managers' compensation through stock prices.

□ **Stock options.** Consider now stock option plans with deferred realization.

Definition 6. Incentive contract class $B^{\underline{P}}$ ($ICB^{\underline{P}}$): In each period t, the manager receives the right to buy a number γ_i of shares at a predetermined price $\underline{P}_i$, both of which are constant across time periods, *after* period-t profits are paid out as dividends.

Because managers get paid after the distribution of dividends, the value of the shares when they cash their stock options does not incorporate present profits π_i^t. Also, consider stock option plans such that managers can exercise the options only some time after retirement. Again, I assume that in every period the manager faces a constant probability $(1 - \eta)$ of leaving the firm.

Definition 7. Incentive contract class $C^{\underline{P}}$ ($ICC^{\underline{P}}$): In each period the manager receives a flat wage, which we normalized to zero, and in the period after he stops working for the firm, say τ periods after he started, the manager receives a number of stock options $\gamma_i(1 + r)^\tau$—where $1/1 + r = \delta$—with a strike price $\underline{P}_i$ and γ_i and $\underline{P}_i$ constant across time periods.

Then I can state what follows.

Proposition 5. Suppose the repeated oligopoly game is played by managers under incentive contracts in the class $ICB^{\underline{P}}$ or $ICC^{\underline{P}}$, with $\underline{P}_i < (\pi_i^M/(1 - \delta))/\varphi_i \ \forall i$. Then the joint monopoly collusive agreement can be supported in a subgame-perfect equilibrium at any level of the discount factor.

The intuition behind this result is fully analogous to that behind Proposition 4.

5. On the choice and length of managerial contracts

■ In the previous sections, managers were assumed to have an infinite horizon, as the firm, and to be under stock-related compensation. Although managers do tend to stay with one firm for long periods, in reality explicit managerial contracts are rarely lifelong. Furthermore, in reality owners are free to change managerial incentives in time. In this section I show that the results in the previous sections apply when explicit managerial contracts are short term and shareholders can change managerial incentives in time. To do this, I assume that managerial contracts last a finite number of periods T, and that owners can choose between profit-related or stock-related incentives. Every T periods, owners must decide whether to reconfirm the current manager and his incentive contract or to replace one or both of them. This situation can be modelled as a repeated game whose stage game is composed of several consecutive steps. To simplify, I focus on symmetric stationary collusive agreements and assume that none of them are supportable when profit-maximizing managers are in control. To further simplify exposition, I assume that all managers' explicit contracts last the same number of periods T and are signed (and expire) simultaneously. It will become clear below that the results of this section are not dependent on these simplifying assumptions.

The stage game of the oligopoly supergame will now be composed of $T + 1$ steps. The timing of a stage game beginning in period t will be as follows.

Stage game t.

Step 1: Owners hire managers and choose their incentive contracts.

Steps 2 to $T + 1$: All players observe the outcome of the previous step, then managers choose output levels.

In other words, in each stage game in step 1, each of the owners simultaneously chooses a manager and his incentive contract for the T following periods. From step 2 on, the managers choose simultaneously output in T consecutive static Cournot market interactions. This means that a new stage game will only begin every $T + 1$ periods.

I can now state the following result.

Proposition 6. Any collusive agreement supportable by managers under lifelong stock-related incentive contracts can be supported as a subgame-perfect-equilibrium outcome of the infinitely repeated delegation game that has the multistage game above as its stage game.

That is, the pro-collusive effects identified in Sections 3 and 4 apply when owners are free to choose profit-related incentives instead of stock-related ones and when explicit managerial contracts are short term. This is because even though explicit managerial contracts last a finite number of periods, owners are free to agree with their managers on *implicit* employment contracts, which are long term by definition (MacLeod and Malcomson, 1989; Carmichael, 1989). In this framework, an owner's choice of a different explicit managerial contract than the agreed stock-related one that leads the manager to sustain collusion is considered a deviation and punished with the interruption of cooperation.

On the side of managers, the negative effects of stock-based incentives on short-run gains from deviations highlighted in Sections 3 and 4 remain when explicit managerial contracts are short term. Regarding owners, they have no incentive whatsoever to deviate by reneging on the implicit contract to reconfirm the manager or by changing the agreed explicit stock-related incentives, since changes of management or incentives are observable and other firms' managers can react before any short-run gain from deviation can be realized. This is also why the proposition can easily be proved to hold when explicit managerial contracts are not signed (do not expire) simultaneously or have different durations.

Note that the converse of the proposition is not true. In fact, the incentive-compatibility conditions for the self-enforcing implicit contracts with short-term explicit contracts that replicate the results in the previous sections can be less stringent than the incentive-compatibility conditions with long-term explicit contracts.

Corollary 2. If managers have stock-related incentives in the class ICA or in the class ICA$^{\underline{P}}$ with $\underline{P}_i < (\pi_i^N/(1 - \delta))/\varphi_i \; \forall i$, then the shorter the length T of the explicit managerial contracts, the smaller the minimum discount factor at which any collusive agreement can be supported (and, for a given discount factor, more profitable agreements become sustainable) in a subgame-perfect equilibrium in the delegation supergame.

Again, owners have no incentives to renege on the implicit managerial contracts, while the negative effect of stock-based incentives on managers' short-run gains from deviations highlighted in Section 3 does not depend on the length of the explicit managerial contracts. Moreover, with short-term contracts at the end of the stage in which a manager deviates, the manager is fired and kept at his reservation wage forever after. Since implicit labor contracts are enforced by leaving each party a stake of the expected (collusive) surplus the employment relation generates (MacLeod and Malcomson, 1989), the threat of termination, with the loss of future rents it implies for the manager, has an additional pro-collusive effect that adds to that identified in Section 3. Because termination is closer in time, the shorter the length of explicit managerial contracts, the stronger the overall pro-collusive effect.

6. Extensions and discussion

■ **Alternative specifications of the model.** *Dividend policy.* The pro-collusive effects identified in Sections 3 and 4 are driven by stock-based managerial incentives being paid for several consecutive periods (ICA, ICB) or being deferred (ICB, ICC), and by the assumption that a firm's stock price depends only the discounted flow of future profits. The time structure of stock-related incentives in my model reflects the evidence on most real-world arrangements (Kole, 1997). The assumption that the stock price is only a function of future profits is standard in finance, and it is appropriate in this model where, with neither uncertainty nor investment, managers have no justification for retaining profits in the firm. However, this last assumption is not necessary for any of the pro-collusive effects I identified. All the results apply when managers are free to retain any fraction of profits in the firm, as long as the total amount of retained profits they can accumulate is bounded away from infinity. By substituting in conditions (2), (4), (5), and in the equations in the proofs, one can verify that the only case in which the behavior of managers under stock-based incentives corresponds to that of profit-maximizing managers or owners—so that the pro-collusive effects disappear—is when managers can accumulate an infinite amount of retained profits within the firm (a proof is available from the author upon request). Of course, the larger the amount of profits managers can retain, the weaker the pro-collusive effects of stock-related incentives.

Market structure. The results are also robust to changes in modelling assumptions about market structure. It is straightforward to check that they apply to repeated oligopolies other than the Cournot type. All results and proofs are stated using only profit streams π_i^N, π_i^M, $\hat{\pi}_i^M$, etc., with no direct reference to the specific strategic variables used in the product market. I can reinterpret the profit stream as deriving from any other repeated oligopoly (for example, setting $\pi_i^N = 0$ in the case of Bertrand competition), and all proofs continue to hold.

□ **Profit sharing and managerial ownership.** I have focused on liquid incentives related to stock price. What if owners choose managerial contracts that also incorporate a profit-sharing component, or require managers to retain firm's shares received as bonuses?

It is straightforward to check that any additional profit-sharing component leads managers to behave more like owners; it dilutes the pro-collusive effect of stock-based incentives without bringing any countervailing benefit. Because of this, in my model the choice to have a profit-sharing incentive component besides stock-related incentives is always dominated (see Spagnolo, 1998a).

Analogous reasoning applies when managers are required to keep in their portfolios the shares they get each period as bonuses. If managers under contracts in the classes ICA or ICB keep the shares they receive each period, they will in time own an increasing fraction of the firm. This leads them to receive a larger and larger share of the profits realized in each period as dividends, with an effect on product-market behavior identical to that of a profit-sharing incentive component increasing in time. The more shares the manager owns, the more dividends he receives, the more he behaves like an owner, the smaller the set of collusive agreements he is willing to support, and the higher the minimum discount rate at which he is willing to stick to any given collusive agreement. Summarizing:

Remark 1. Profit-sharing incentives, restrictions on the resale or transfer of firm shares received as bonuses, and, more generally, managerial ownership dilute the pro-collusive effects of stock-related compensation plans.

□ **Incentives linked to sales.** In my oligopoly supergame, owners can enforce tacit agreements to restrict output. Because in collusive equilibria output is given by the tacit agreement, colluding owners cannot gain strategic advantages (such as reductions in competing firms' output) by delegating control to managers under aggressive Fershtman-Judd–Sklivas-type incentives linked to sales revenue. Incentives linked to sales, though, may still play a role since they may affect owners' gains from deviations and payoffs in the punishment phase.

Consider the case analyzed in Section 5, with explicit managerial contracts of any time length T, and suppose owners can also choose Fershtman-Judd–Sklivas-type incentive schemes linear in profits and sales. In step 1 of any stage game, an owner who expects other owners to choose pro-collusive stock-based managerial incentives may wish to deviate by choosing an aggressive Fershtman-Judd–Sklivas-type managerial contract increasing with sales. In the remainder of the supergame, collusion would not be sustained, but the deviating owner would enjoy Stackelberg profits π_i^S for the first T periods after the deviation, and of course $\pi_i^S > \pi_i^N$. Clearly, when $\pi_i^S < \pi_i^M$ the joint monopoly outcome is still supportable in equilibrium, since if owners expect other owners to choose the agreed stock-related incentives that lead managers to support the joint monopoly agreement, they lose strictly by deviating and choosing aggressive Fershtman-Judd–Sklivas-type incentives, whatever T and δ are. More generally, whether π_i^S is $>$, $=$, or $<$ than π_i^M, any agreement to delegate control to managers under stock-related incentives leading to a collusive outcome with per-period profits π_i^A remains supportable as long as $\pi_i^A > (1 - \delta^T)\pi_i^S + \delta^T \pi_i^{FJS}$, where π_i^{FJS} denotes profits at the Nash equilibrium of the Fershtman-Judd–Sklivas delegation game and $\pi_i^{FJS} < \pi_i^N$.[9] For small-enough T, this condition is satisfied even for less profitable agreements, since the owners' short-run gains from deviation generated by the opportunity to choose Fershtman-Judd–Sklivas-type incentives are outweighed by the lower profits this opportunity induces during the subsequent noncooperative phase.

Finally, if owners can choose "collusive" stock-based incentives and "aggressive" Fershtman-Judd–Sklivas-type incentives simultaneously, collusion can be further stabilized. To see this, consider a duopoly and the possibility of such "mixed" compensation contracts. Suppose the incentive part I_i of a manager's per-period compensation can be composed of a Fershtman-Judd–Sklivas-type incentive scheme linear in per-period profits and in sales revenue (denoted by S_i), plus an additional stock-related bonus plan as in the previous sections. That is,

$$I_i = \rho_i(\alpha_i \pi_i + (1 - \alpha_i)S_i) + (1 - \rho_i)\mathrm{IC}_i(\mathrm{P}_i),$$

where $\mathrm{IC}_i(\mathrm{P}_i)$ can be chosen from the classes defined in Sections 3 and 4. Let α^{FJS} denote the Nash equilibrium level of the parameter α in the classical Fershtman-Judd–Sklivas two-stage duopoly model. We get immediately the following result.

Proposition 7. Even when $\pi_i^S > \pi_i^M$, and whatever T and δ are, any collusive agreement A delivering per-period profits π_i^A can be implemented by a mixed managerial contract with $\alpha = \alpha^{FJS}$, $\mathrm{IC}_i(\mathrm{P}_i) = \mathrm{ICB}^{\underline{P}}$ (or $\mathrm{ICC}^{\underline{P}}$, or $\mathrm{ICA}^{\underline{P}}$ when (5) is satisfied), with $\underline{P}_i > \pi^{FJS}$, and $0 < \rho < 1$.

A formal proof would be analogous to that of Proposition 6 but is not needed, the logic behind the proposition being straightforward. The pro-collusive effects of stock options identified in Sections 3 and 4 remain when these are part of a more complex managerial incentive scheme. In addition, when owners use the mixed contract described above, if in step 1 of a stage game an owner deviates by setting $\rho_i = 1$, the

[9] It is $\pi_i^{FJS} < \pi_i^N$ because with quantity competition, attempts to gain a strategic advantage through precommitment offset one another (see Fershtmann and Judd, 1987, or Sklivas, 1987).

competing manager reacts already in step 2 by maximizing the Fershtman-Judd–Sklivas-type part of his incentive scheme only, since his options are valueless whatever he does. Then, already from step 2, instead of π_i^S the deviating manager obtains π_i^{FJS}. Therefore this mechanism, which is reminiscent of the one in Reitman (1993), further stabilizes collusion by ensuring that a deviating owner incurs a direct loss in the same period in which he deviates.

□ **Demand uncertainty.** Some authors emphasize results obtained with demand uncertainty, both because uncertainty makes the model more realistic and because it leaves a nontrivial role to managers. The managers' task is then to observe the realization of demand, which occurs after the delegation phase, and then choose output using that information (Fershtman and Judd, 1987; Reitman, 1993).

Let θ denote the stochastic component of demand, and assume θ to be independently and identically distributed in time and its distribution to be common knowledge among agents. As in Rotemberg and Saloner (1986), in my model with demand uncertainty the expected losses from the punishment phase that disciplines the collusive agreement will be constant in time, while short-run gains from deviation will change together with the realization of the state of the world θ. Then, when the discount factor binds, most profitable collusive agreements must be conditioned on the per-period realization of the shock θ. Whether the supergame is played by owners or by managers, players can agree on a ''collusive rule'' $q^A(\theta) = (q_1^A(\theta), \ldots, q_n^A(\theta))$ mapping states of the world into firms collusive output levels, and eventually into profits. The rule can be chosen to ensure, given agents' discount factor and the expected punishment for deviations, that for each realization of θ the prescribed collusive output levels are such that the incentive constraint is satisfied for all players. It is simple to check that the introduction of demand uncertainty leaves the results unchanged. Demand uncertainty adds to strategic uncertainty from the *ex ante* point of view, so that we must substitute π_i^N, π_i^A, $\hat{\pi}_i^A$, π_i^M, . . . , etc., with the corresponding expected values $E_\theta[\pi_i^N(\theta)]$, $E_\theta[\pi_i^A(\theta)]$, $E_\theta[\hat{\pi}_i^A(\theta)]$, $E_\theta[\pi_i^M(\theta)]$, . . . , etc., in agents' incentive constraints. Also, when owners use stock options they will now choose a strike price conditional on the state of demand $\underline{P}_i(\theta)$, if θ can be contracted upon, or otherwise keep $\underline{P}_i$ below $\min_\theta \{\pi_i^M(\theta)\}$ in order to make collusion supportable in all states of demand. However, the logic behind my results goes through.

□ **Alternative punishment strategies.** I assumed that firms sustain collusive agreements by the threat of reverting to the static Nash equilibrium of the oligopoly game forever. Unrelenting trigger strategies are widely used in the literature because they satisfy the requirement of subgame perfection and are easy to handle. However, this kind of punishment is not optimal in repeated Cournot oligopolies (Abreu, 1986) and may be subject to *ex post* renegotiation, which would undermine their credibility (Farrell and Maskin, 1989; Bernheim and Ray, 1989).

It is easy to check that all the results continue to hold when the threat used to enforce collusion is to revert to the static Nash equilibrium only for a finite number of periods, for example because the strength of the punishment is bounded by a finite cost of renegotiation (as in McCutcheon (1997); see also Blume (1994)).

More generally, the results concerning deferred stock-related incentives depend on managers being unable to capture any short-run gains from deviation. Therefore, the results in Section 4 apply independent of what punishment strategies are used. Even in the case of ICC-type incentives, for which there is the chance that a punishment phase of finite length has passed at the time when the incentives are paid, the pro-collusive effect is independent of the shape of the punishment phase as long as the date when the manager leaves the firm is uncertain.[10]

What if managers have contracts in the class ICA or $\text{ICA}\underline{p}$ and there are no renegotiation costs? In my working paper (Spagnolo, 1998a) I analyze the case of long-term stock-option plans and find that the results in Section 3 can be extended both to the case of Abreu's (1986) two-phase optimal punishments and to that of van Damme's (1989) ''repentance'' renegotiation-proof strategies.

□ **Renegotiation of managerial contracts.** In the Introduction I argued that stock-related incentives are less subject than other types of incentives to secret renegotiation, because they require shareholders' approval given in public shareholders' meetings. But even if we assume completely concentrated ownership, so that public shareholders' meetings are not required to renegotiate managers' compensation, the results of the model would still be of interest.

This is because the *costs* of secret renegotiation may be substantial for owners; and renegotiation costs give commitment value to managerial incentives. There will typically be direct costs of the bilateral bargaining process between managers and owners, even if there are no information asymmetries (Anderlini and Felli, 1998). When third parties (for example, debtholders) have seats on the board, the bargaining process becomes trilateral and bargaining costs increase. Moreover, interlocked directors and large finance providers with industrywide interests will oppose any renegotiation of a managerial contract that leads to a market war (see Spagnolo, 1996, 1998b).

7. Concluding remarks

■ I am not arguing here that the effect on tacit collusion is the only or the main force driving firms' adoption of managerial compensation plans related to stock price. As in most previous work on the strategic effects of delegation, to make the model tractable I had to abstract from many important issues, particularly from that of managerial moral hazard (just as most of the literature on moral hazard abstracts from the strategic effects of incentive contracts). When managers' moral hazard is brought into the picture, many other beneficial effects of these incentives emerge.

However, I believe that in the imperfectly competitive real world, the pro-collusive effect of these incentives may be one important reason behind their success. In the end, shareholders are satisfied when their managers' incentive schemes lead to higher stock prices, regardless of whether this is achieved through higher effort or more effective collusion.

Appendix

■ Proofs of Propositions 1–6 and of Corollaries 1–2 follow.

Proof of Proposition 1. At $\delta = \underline{\delta}^A$ condition (1) is satisfied as an equality and

$$\frac{\pi_i^A}{1-\underline{\delta}^A} = \hat{\pi}_i^A + \frac{\underline{\delta}^A}{1-\underline{\delta}^A}\pi_i^N, \qquad \Rightarrow \frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right) = \frac{1}{\varphi_i}\left(\hat{\pi}_i^A + \frac{\underline{\delta}^A}{1-\underline{\delta}^A}\pi_i^N\right).$$

Substituting from the last equality into (2) I obtain

[10] Of course if the punishment phase lasts one period only, as in Abreu's (1986) two-phase optimal punishments, and the ICC-type incentive is deferred for more than one period with certainty, then neither a deviation nor the punishment affect manager's compensation, and we are led to the owners' incentive-compatibility condition. However, in this case owners can simply choose incentives in the classes ICA and ICB.

$$\frac{1}{1-\underline{\delta}^A} f_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right)\right] > f_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right)\right] + \frac{\underline{\delta}^A}{1-\underline{\delta}^A} f_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^N}{1-\underline{\delta}^A}\right)\right],$$

which after a few algebraic manipulations becomes

$$f_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right)\right] > f_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^N}{1-\underline{\delta}^A}\right)\right],$$

which is always satisfied. Because the inequality is strict, by continuity, perturbing the discount factor around $\underline{\delta}^A$ we can find a continuum of discount factors lower than $\underline{\delta}^A$ at which this condition is satisfied but (1) is not. This reasoning applies to any stationary collusive agreement A and to each firm i. Statement (i) follows.

Conversely, given agents' discount factor, at the most collusive agreement that owners can support, delivering $\pi_i^{\overline{A}}$, we have $(\pi_i^{\overline{A}}/(1-\delta))/\varphi_i = 1/\varphi_i[\hat{\pi}_i^{\overline{A}} + (\delta/1-\delta)\pi_i^N]$; substituting into condition (2) and simplifying we obtain

$$f_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^{\overline{A}}}{1-\delta}\right)\right] > f_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^N}{1-\delta}\right)\right],$$

which is always true. By continuity, perturbing profits around $\pi_i^{\overline{A}}$ we can find a continuum of higher collusive profit streams that satisfy this condition but not condition (1). This reasoning applies to any stationary collusive agreement and to each firm i. Statement (ii) follows. *Q.E.D.*

Proof of Proposition 2. Substituting for stock prices into (4) I obtain

$$\frac{1}{1-\delta}\frac{\alpha_i K}{\varphi_i}\frac{\pi_i^A}{1-\delta} > \alpha_i \frac{\hat{\pi}_i^A + \delta\dfrac{K\pi_i^N}{1-\delta}}{\varphi_i\left(1+\dfrac{1}{\varphi_i}\dfrac{\alpha_i}{\varphi_i}\right)} + \frac{\delta}{1-\delta}\frac{\alpha_i K}{\varphi_i}\frac{\pi_i^N}{1-\delta},$$

or, equivalently,

$$\frac{\pi_i^A}{1-\delta} > \frac{(1-\delta)\left(\hat{\pi}_i^A + \delta\dfrac{K\pi_i^N}{1-\delta}\right)}{K\left(1+\dfrac{1}{\varphi_i}\dfrac{\alpha_i}{\varphi_i}\right)} + \frac{\delta\pi_i^N}{1-\delta}.$$

This inequality is less stringent than (1), so that it is satisfied at $\delta < \underline{\delta}^A$ and at $\pi^A > \pi^{\overline{A}}$ and the statements hold, if

$$\frac{(1-\delta)\left(\hat{\pi}_i^A + \delta\dfrac{K\pi_i^N}{1-\delta}\right)}{K\left(1+\dfrac{1}{\varphi_i}\dfrac{\alpha_i}{\varphi_i}\right)} < \hat{\pi}_i^A,$$

or, equivalently, if

$$\frac{\delta\pi_i^N}{\left(1+\dfrac{1}{\varphi_i}\dfrac{\alpha_i}{\varphi_i}\right)} < \hat{\pi}_i^A - \frac{(1-\delta)\hat{\pi}_i^A}{K\left(1+\dfrac{1}{\varphi_i}\dfrac{\alpha_i}{\varphi_i}\right)},$$

which reduces to

$$\delta\pi_i^N < \hat{\pi}_i^A + \hat{\pi}_i^A\frac{1}{\varphi_i}\frac{\alpha_i}{\varphi_i} - \hat{\pi}_i^A(1-\delta) - \hat{\pi}_i^A\frac{1}{\varphi_i}\frac{\alpha_i}{\varphi_i} \Rightarrow \pi_i^N < \hat{\pi}_i^A,$$

which is always true. *Q.E.D.*

Proof of Proposition 3. Manager i's incentive-compatibility constraint is (5). Evaluating (1) at $\delta = \underline{\delta}^A$ I obtain

$$\frac{\pi_i^A}{1-\underline{\delta}^A} = \hat{\pi}_i^A + \frac{\underline{\delta}^A}{1-\underline{\delta}^A}\pi_i^N, \qquad \Rightarrow \frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right) = \frac{1}{\varphi_i}\left(\hat{\pi}_i^A + \frac{\underline{\delta}^A}{1-\underline{\delta}^A}\pi_i^N\right).$$

Substituting from this equality into (5) and simplifying I obtain

$$\gamma_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\underline{\delta}^A}\right) - \underline{P}_i\right] > \max\left\{\gamma_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^N}{1-\underline{\delta}^A}\right) - \underline{P}_i\right], 0\right\}.$$

By inspection, for any strike price $\underline{P}_i < (\pi_i^A/(1-\underline{\delta}^A))/\varphi_i$ and number of options $\gamma_i \neq 0$ this condition holds as a *strict* inequality. By continuity, perturbing the discount factor around $\underline{\delta}^A$ we can find a continuum of discount factors lower than $\underline{\delta}^A$ (of more collusive agreement, i.e., $\pi_i \geq \pi_i^A$) at which such a condition is satisfied but (1) is not. This reasoning applies to any stationary collusive agreement A and to each firm i. This proves statement (i).

Statement (ii) follows from the converse argument (see case (ii) in the proof of Proposition 1). *Q.E.D.*

Proof of Corollary 1. Consider first the case $(\pi_i^N/(1-\delta))/\varphi_i \leq \underline{P}_i < (\pi_i^A/(1-\delta))/\varphi_i$. Condition (5) becomes

$$\frac{1}{1-\delta}\gamma_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\delta}\right) - \underline{P}_i\right] > \gamma_i\left[\frac{1}{\varphi_i}\left(\hat{\pi}_i^A + \frac{\delta\pi_i^N}{1-\delta}\right) - \underline{P}_i\right],$$

or, equivalently,

$$\delta\left[\frac{1}{\varphi_i}\left(\hat{\pi}_i^A + \frac{\delta\pi_i^N}{1-\delta}\right) - \underline{P}_i\right] > \frac{1}{\varphi_i}\left(\hat{\pi}_i^A + \frac{\delta\pi_i^N}{1-\delta}\right) - \frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\delta}\right).$$

Then, a manager under an ICA$^{\underline{P}}$-type contract with strike price $\underline{P}_i$ is willing to support any given collusive agreement A as long as

$$\delta > \underline{\delta}^A_{ICA^{\underline{P}}} = \frac{\frac{1}{\varphi_i}\left(\hat{\pi}_i^A + \frac{\underline{\delta}^A_{ICA^{\underline{P}}}\pi_i^N}{1-\underline{\delta}^A_{ICA^{\underline{P}}}}\right) - \frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\underline{\delta}^A_{ICA^{\underline{P}}}}\right)}{\frac{1}{\varphi_i}\left(\hat{\pi}_i^A + \frac{\underline{\delta}^A_{ICA^{\underline{P}}}\pi_i^N}{1-\underline{\delta}^A_{ICA^{\underline{P}}}}\right) - \underline{P}_i}.$$

By inspection, $\underline{\delta}^A_{ICA^{\underline{P}}}$ is independent of γ_i and is increasing with $\underline{P}_i$. Analogously, the upper bound of the collusive profit streams supportable by a manager under ICA$^{\underline{P}}$-type contracts is $\pi_i^{\overline{ICA^{\underline{P}}}}$, where

$$\frac{1}{\varphi_i}\left(\frac{\pi_i^{\overline{ICA^{\underline{P}}}}}{1-\delta}\right) - (1-\delta)\frac{1}{\varphi_i}\left(\hat{\pi}_i^{\overline{ICA^{\underline{P}}}} + \frac{\delta\pi_i^N}{1-\delta}\right) = \delta\underline{P}_i.$$

Take the upper bound $\pi^{\overline{ICA^{\underline{P}'}}}$ that satisfies the equality above at a given strike price $\underline{P}_i'$. A reduction in $\underline{P}_i$ makes the condition satisfied as a strict inequality, moving the upper bound to a higher profit level. So $\pi_i^{\overline{ICA^{\underline{P}}}}$ is a decreasing function of $\underline{P}_i$.

Consider now the case of $\underline{P}_i < (\pi_i^N/(1-\delta))/\varphi_i$. The incentive-compatibility condition becomes

$$\frac{1}{1-\delta}\gamma_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\delta}\right) - \underline{P}_i\right] > \gamma_i\left[\frac{1}{\varphi_i}\left(\hat{\pi}_i^A + \frac{\delta\pi_i^N}{1-\delta}\right) - \underline{P}_i\right] + \frac{\delta}{1-\delta}\gamma_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^N}{1-\delta}\right) - \underline{P}_i\right].$$

The minimum level of the discount factor at which the manager can support collusion becomes

$$\underline{\delta}^A_{ICA^{\underline{P}}} = \frac{\frac{1}{\varphi_i}\left(\hat{\pi}_i^A + \frac{\underline{\delta}^A_{ICA^{\underline{P}}}}{1-\underline{\delta}^A_{ICA^{\underline{P}}}}\pi_i^N\right) - \frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\underline{\delta}^A_{ICA^{\underline{P}}}}\right)}{\frac{1}{\varphi_i}\left(\hat{\pi}_i^A + \frac{\underline{\delta}^A_{ICA^{\underline{P}}}}{1-\underline{\delta}^A_{ICA^{\underline{P}}}}\pi_i^N\right) - \frac{1}{\varphi_i}\left(\frac{\pi_i^N}{1-\underline{\delta}^A_{ICA^{\underline{P}}}}\right)},$$

and the condition that identifies the most collusive agreement supportable at the given discount factor $\pi_i^{\overline{ICA^{\underline{P}}}}$ becomes

$$\frac{1}{\varphi_i}\left(\frac{\pi_i^{\overline{ICA}^{\underline{P}}}}{1-\delta}\right) = (1-\delta)\frac{1}{\varphi_i}\left(\hat{\pi}_i^{\overline{ICA}^{\underline{P}}} + \frac{\delta\pi_i^N}{1-\delta}\right) + \delta\frac{1}{\varphi_i}\left(\frac{\pi_i^N}{1-\delta}\right).$$

The last two equalities are both independent of γ_i and $\underline{P}_i$. All this holds for every firm i and the statement follows. *Q.E.D.*

Proof of Proposition 4. Consider first the class ICB. The incentive-compatibility condition for a stationary collusive agreement A to be respected by the manager of firm i under an ICB-type contract is

$$\frac{1}{1-\delta} f_i\left[\frac{1}{\varphi_i}\left(\frac{\delta\pi_i^A}{1-\delta}\right)\right] > f_i\left[\frac{1}{\varphi_i}\left(\frac{\delta\pi_i^N}{1-\delta}\right)\right] + \frac{\delta}{1-\delta} f_i\left[\frac{1}{\varphi_i}\left(\frac{\delta\pi_i^N}{1-\delta}\right)\right],$$

which is *always* satisfied, at any discount factor δ, agreement A, and firm i.

Consider now contracts in the class ICC. The expected flow of earnings for the manager of firm i in any period t in which he is running the firm is

$$\delta(1-\eta)(1+r)f_i(P_i^{t+1}) + \delta^2\eta(1-\eta)(1+r)^2 f_i(P_i^{t+2}) + \delta^3\eta^2(1-\eta)\gamma_i(1+r)^3 f_i(P_i^{t+3}) + \ldots,$$

which reduces to

$$(1-\eta)\gamma_i \sum_{\tau=1}^{\infty} \eta^{\tau-1} f_i(P_i^{t+\tau}).$$

As long as the manager sticks to a stationary collusive agreement delivering per-period profits π_i^A, we have $P^{t+\tau} = (\pi_i^A/(1-\delta))/\varphi_i \ \forall\tau > 0$. If the manager deviates in any period t, we have

$$P^{t+\tau} = (\pi_i^N/(1-\delta))/\varphi_i \quad \forall\tau > 0.$$

Because $(\pi_i^A/(1-\delta))/\varphi_i > (\pi_i^N/(1-\delta))/\varphi_i$ is always satisfied, whatever the discount factor δ the manager always finds it convenient not to deviate from the agreement. This applies to any agreement A and firm i, and the statement follows. *Q.E.D.*

Proof of Proposition 5. It is analogous to that of Proposition 4 and can be found in Spagnolo (1998a).

Proof of Proposition 6. Consider the following strategy profile for the delegation supergame.

Each owner's strategy:

''Delegate control to a manager under a stock-related explicit contract among those defined in Sections 3 and 4, of finite length T, such that if the same contract had infinite length (if $T \to \infty$), the manager would be willing to support the collusive agreement A delivering per-period profits π_i^A, and such that at least at this collusive equilibrium the manager is paid above his reservation wage; at the beginning of each of the following stage games (in periods $t + T$, $t + 2T$, etc.), reconfirm the manager and the contract for one more stage if all other owners have done so in the past and no manager has ever deviated from equilibrium strategies; hire a profit-maximizing manager at his reservation wage forever otherwise.''

Each manager's strategy:

''Respect the collusive agreement A at all steps of each stage game as long as all owners have delegated/reconfirmed managers with the incentive contracts described above in each past stage game and no manager has ever deviated from the collusive agreement; maximize the firm's static profits forever otherwise.''

It is easy to verify that this strategy profile is a subgame-perfect equilibrium, independent of the length of the managerial contracts and of the discount factor (a complete proof is in Spagnolo (1998a)). *Q.E.D.*

Proof of Corollary 2. Consider the case of managers under ICA contracts of length T, and maintain the previous proof's normalization of managers' reservation wage to zero. The managers will stick to an agreement A as long as

$$\frac{1}{1-\delta} f_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^A}{1-\delta}\right)\right] > f_i\left[\frac{1}{\varphi_i}\left(\hat{\pi}_i^A + \frac{\delta\pi_i^N}{1-\delta}\right)\right] + \frac{\delta(1-\delta^{T-1})}{1-\delta} f_i\left[\frac{1}{\varphi_i}\left(\frac{\pi_i^N}{1-\delta}\right)\right].$$

Because $[\delta(1-\delta^{T-1})]/1-\delta$ is increasing in T, the right-hand side of the inequality is increasing in T. It follows that the condition becomes more stringent the larger T is. Analogous reasoning holds for contracts in the class $\text{ICA}^{\underline{P}}$ with $\underline{P}_i < (\pi_i^N/(1-\delta))/\varphi_i \ \forall i$. *Q.E.D.*

References

ABREU, D. "Extremal Equilibria of Oligopolistic Supergames." *Journal of Economic Theory,* Vol. 39 (1986), pp. 191–225.

———. "On the Theory of Infinitely Repeated Games with Discounting." *Econometrica,* Vol. 56 (1988), pp. 383–396.

AGGARWAL, R. AND SAMWICK, A. "Executive Compensation, Strategic Competition, and Relative Performance Evaluation." *Journal of Finance,* Vol. 54 (1999), pp. 1999–2043.

ANDERLINI, L. AND FELLI, L. "Costly Bargaining and Renegotiation." Mimeo, London School of Economics, 1998.

AUMANN, R.J. AND SHAPLEY, L.S. "Long-Term Competition—A Game Theoretic Analysis." University of California-Los Angeles Department of Economics Working Paper no. 676, 1992.

BAUMOL, W.J. "On the Theory of Oligopoly." *Economica,* Vol. 25 (1958), pp. 187–198.

BEBCHUK, L.A. AND STOLE, L.A. "Do Short-Term Objectives Lead to Under- or Overinvestment in Long-Term Projects?" *Journal of Finance,* Vol. 48 (1993), pp. 719–729.

BERNHEIM, D. AND RAY, D. "Collective Dynamic Consistency in Repeated Games." *Games and Economic Behavior,* Vol. 1 (1989), pp. 295–326.

BHAGAT, S., BRICKLEY, J.A., AND LEASE, R.C. "Incentive Effects of Stock Purchase Plans." *Journal of Financial Economics,* Vol. 14 (1985), pp. 195–215.

BIZJAK, J.M., BRICKLEY, J.A., AND COLES, J.L. "Stock-Based Incentive Compensation and Investment Behavior." *Journal of Accounting and Economics,* Vol. 16 (1993), pp. 349–372.

BLUME, A. "Intraplay Communication in Repeated Games." *Games and Economic Behavior,* Vol. 6 (1994), pp. 181–211.

CARMICHAEL, H.L. "Self-Enforcing Contracts, Shirking, and Life Cycle Incentives." *Journal of Economic Perspectives,* Vol. 3 (1989), pp. 65–83.

CYERT, R.M. AND MARCH, J.G. *A Behavioral Theory of the Firm.* Engelwood Cliffs, N.J.: Prentice-Hall, 1963.

DEWATRIPONT, M. "Commitment Through Renegotiation-proof Contracts with Third Parties." *Review of Economic Studies,* Vol. 55 (1988), pp. 377–390.

FARRELL, J. AND MASKIN, E. "Renegotiation in Repeated Games." *Games and Economic Behavior,* Vol. 1 (1989), pp. 327–360.

FERSHTMAN, C. "Managerial Incentives as a Strategic Variable in Duopolistic Environment." *International Journal of Industrial Organization,* Vol. 3 (1985), pp. 245–253.

——— AND JUDD, K.L. "Equilibrium Incentives in Oligopoly." *American Economic Review,* Vol. 77 (1987), pp. 927–940.

———, ———, AND KALAI, E. "Observable Contracts: Strategic Delegation and Cooperation." *International Economic Review,* Vol. 32 (1991), pp. 551–559.

FRIEDMAN, J.W. "A Non-Cooperative Equilibrium for Supergames." *Review of Economic Studies,* Vol. 38 (1971), pp. 1–12.

FUDENBERG, D. AND MASKIN, E. "The Folk Theorem in Repeated Games with Discounting or Incomplete Information." *Econometrica,* Vol. 54 (1986), pp. 533–554.

GREEN, E. AND PORTER, R. "Noncooperative Collusion Under Imperfect Price Information." *Econometrica,* Vol. 52 (1984), pp. 87–100.

HALL, B.J. AND LIEBMAN, J.B. "Are CEOs Really Paid Like Bureaucrats?" *Quarterly Journal of Economics,* Vol. 113 (1998), pp. 653–691.

HOLMSTRÖM, B. AND TIROLE, J. "Market Liquidity and Performance Monitoring." *Journal of Political Economy,* Vol. 101 (1993), pp. 678–709.

JACKSON, M. AND LAZEAR, E. "Stock, Options, and Deferred Compensation." In R. Ehrenberg, ed., *Research in Labor Economics,* Vol. 12. Greenwich, Conn.: JAI Press, 1991.

JENSEN, M.C. AND MECKLING, W. "Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics,* Vol. 3 (1976), pp. 305–360.

——— AND MURPHY, K. "Performance Pay and Top-Management Incentives." *Journal of Political Economy,* Vol. 98 (1990), pp. 225–264.

KAPLAN, S.N. "Top Executive Rewards and Firm Performance: A Comparison of Japan and the United States." *Journal of Political Economy,* Vol. 102 (1994a), pp. 510–546.

———. "Top Executives, Turnover, and Firm Performance in Germany." *Journal of Law, Economics and Organization,* Vol. 10 (1994b), pp. 142–159.

KATZ, M.L. "Game-Playing Agents: Unobservable Contracts as Precommitments." *RAND Journal of Economics,* Vol. 22 (1991), pp. 307–328.

KOLE, S.R. "The Complexity of Compensation Contracts." *Journal of Financial Economics,* Vol. 43 (1997), pp. 79–104.

LAING, J.R. ''Big Mac Wednesday. McDonald's Price War Battle Plan Casts a Pall Over Fast-Food Stocks.'' *Barron's,* March 3, 1997, p. 14.

MACLEOD, W.B. AND MALCOMSON, J.M. ''Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment.'' *Econometrica,* Vol. 57 (1989), pp. 447–480.

MARRIS, R. *The Economic Theory of Managerial Capitalism.* New York: Macmillan, 1964.

MCCUTCHEON, B. ''Do Meetings in Smoke-Filled Rooms Facilitate Collusion?'' *Journal of Political Economy,* Vol. 105 (1997), pp. 330–350.

MURPHY, K. ''Corporate Performance and Managerial Remuneration: An Empirical Analysis.'' *Journal of Accounting and Economics,* Vol. 7 (1985), pp. 11–42.

NARAYANAN, M.P. ''Managerial Incentives for Short-Term Results.'' *Journal of Finance,* Vol. 40 (1985), pp. 1469–1484.

POLO, M. AND TEDESCHI, P. ''Managerial Contracts, Collusion and Mergers.'' *Ricerche Economiche,* Vol. 46 (1992), pp. 281–302.

REITMAN, D. ''Stock Options, and the Strategic Use of Managerial Incentives.'' *American Economic Review,* Vol. 83 (1993), pp. 513–524.

ROTEMBERG, J. AND SALONER, G. ''A Supergame-Theoretic Model of Price Wars During Booms.'' *American Economic Review,* Vol. 76 (1986), pp. 390–407.

RUBINSTEIN, A. ''Equilibrium in Supergames with the Overtaking Criterion.'' *Journal of Economic Theory,* Vol. 21 (1979), pp. 1–9.

SCHOLES, M. ''Stock and Compensation.'' *Journal of Finance,* Vol. 46 (1991), pp. 803–823.

SIMON, H. *Administrative Behavior,* 2d ed. New York: Macmillan, 1957.

SKLIVAS, S.D. ''The Strategic Choice of Managerial Incentives.'' *RAND Journal of Economics,* Vol. 18 (1987), pp. 452–458.

SPAGNOLO, G. ''Ownership, Control, and Collusion.'' Working Paper in Economics and Finance no. 139, Stockholm School of Economics, 1996 (available at http://www.ssrn.com/).

———. ''Shareholder-Value Maximization and Tacit Collusion.'' Working Paper in Economics and Finance no. 235, Stockholm School of Economics, 1998a (available at http://swopec.hhs.se/).

———. ''Debt as a (Credible) Collusive Device.'' Working Paper in Economics and Finance no. 243, Stockholm School of Economics, 1998b (available at http://swopec.hhs.se/).

STEIN, J. ''Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior.'' *Quarterly Journal of Economics,* Vol. 104 (1989), pp. 655–669.

VAN DAMME, E. ''Renegotiation-Proof Equilibria in Repeated Prisoners' Dilemma.'' *Journal of Economic Theory,* Vol. 47 (1989), pp. 206–217.

VICKERS, J. ''Delegation and the Theory of the Firm.'' *Economic Journal,* Vol. 95 (1985), pp. 138–147.

WILLIAMSON, O.E. *Managerial Discretion and Business Behavior.* Engelwood Cliffs, N.J.: Prentice-Hall, 1964.