

Networks of Relations, Word-of-Mouth Communication, and Social Capital*

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Abstract

We model networks of relational (or implicit) contracts, exploring how sanctioning power and equilibrium conditions change under different network configurations and information transmission technologies. In our model relations are the links, and the value of the network lies in its ability to enforce cooperative agreements that could not be sustained if agents had no access to other network members' sanctioning power and information. We identify conditions for network stability and in-network information transmission as well as conditions under which stable subnetworks inhibit more valuable larger networks. The model provides formal definitions for individual and communities' "social capital" in the spirit of Coleman and Putnam.

JEL Codes: L13, L29, D23, D43, O17

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1 Introduction

Relational (or implicit) contracts, informal cooperative arrangements sustained by repeated interaction, are a fundamental governance mechanism for most forms of economic and social interaction. When several long-term cooperative relationships link different agents in a group, these agents and their relationships form *networks of relations*. This paper is an attempt to characterize some of their features.

Sociologists have forcefully argued that, by ignoring the networks of social relationships in which economic transactions are "embedded", economists fail to understand important features of the economic process.¹ Like social relations, economic transactions themselves are seldom isolated exchanges. Most often, they are also part of a relationship, episodes of a history of exchanges of various type itself embedded in a network of other economic and social relationships.² This is obviously the case for transactions within organizations – from employment to interactions between units and employees – but also for many of those between organizations, in particular supply relations, including financial ones.

Networks of relational arrangements are not only crucial in developing economies, where explicit contracting is hard: in advanced economic environments, and most prominently in the fast changing one of high-tech industries firms often cooperate to share the high risk and return from their activities. In these industries formal arrangements represent the tip of the iceberg "beneath which lies a sea of informal relations" (Powell et al. 1996). On the one hand, lacking contractibility over the main ingredients – investments into human capital and knowledge transfers – explicit contracts can only play a limited role.³ On the other hand, the need for flexibility linked to the fast changing and highly unpredictable environment make rigid explicit contracts dangerous and vertical integration unattractive. High tech firms therefore often establish informal cooperative agreements with several other firms, and these arrangements link them in a common network of relations.⁴

In the Internet, the maintenance of reciprocal peering agreements between Internet Service

¹The work of Coleman (1988, 1990) is particularly relevant, and that of Granovetter (1985), who writes: "*The embeddedness argument stresses instead the role of concrete personal relations and structures (or "networks") of such relations in generating trust and discouraging malfeasance*" (p. 490). Greif (1993), Spagnolo (1999b), and Casella and Rauch (2002) discuss the importance of social relations and ethnic ties for trade in environments where other enforcement mechanisms are ineffective.

²Macaulay (1963) first drew attention on the crucial role plaid by relationships in the economic process; Klein and Leffler (1981) have stressed the importance of long term firm-customer relationships; cornerstones of the formal theory of implicit contracts are Bull (1987) and MacLeod and Malcomson (1989); Baker et al. (2001), Levin (2003) and Rajo (2003) constitute important recent developments.

³The experimental work of Fehr et al. (2004) nicely documents the overwhelming importance of long term relationships for specific economic transactions.

⁴Saxenian (1994) reports a highly specialized network-like organization within the computer-industry in Silicon Valley. She argues that networks of informal cooperative information-sharing relations play a crucial role for the success of the district in comparison with Route 128, a competing district close to Boston. In her words, "While they competed fiercely, Silicon Valley's producers were embedded in, and inseparable from, these social and technical networks." It is noteworthy that the informal relations reported by Saxenian are not only of value on their own, they are of special value due to their being part of a network of such relations between engineers. Examining the biotechnology industry, Powell et al. (1996) point out that the "development of cooperative routines goes beyond simply learning how to maintain a large number of ties. Firms must learn how to transfer knowledge across alliances and locate themselves in those network positions that enable them to keep pace with the most promising scientific or technological developments."

Providers (ISPs) requires long term cooperative relationships between them, nodes of a network (see e.g. Shin and Weiss, 2004). The interbank market can be seen as a network of long term relationships, where the links that spread contagion among interdependent financial institutions also induce liquid banks to cooperate and privately bail out illiquid ones (see Leitner, 2004). And social networks have been recently shown to have a pervasive, and sometimes negative influence on corporate governance practices (e.g. Kramarz and Thesmar, 2004).

In fact, cooperation is often not for the good: corruption, illegal trade (in drugs, arms and people) and organized crime in general can only rely on relational contracts for the governance of their illegal transactions, which therefore typically take place within networks of tight relations. Collusive agreements to increase prices or restrain output are a common forms of illegal (and common) relational contracts. Multiproduct firms at different levels of the production chain, meeting and cooperating/colluding in different input, geographical, or product markets form networks of relations that may link many apparently distant and unrelated firms, creating pro-collusive *indirect multimarket contact* where no multimarket contact is present.

In this paper we identify equilibrium conditions for different architectures of networks of relations under different informational regimes, paying special attention to differences between circular and non-circular architectures. Most of the dilemmas mentioned earlier, from hold-up situations in specific (legal or illegal) exchanges to cheating on cartel agreements or public good contribution have the strategic features of a Prisoner's Dilemma game, so our basic model is a repeated game in which each agent interacts in generic, asymmetric repeated Prisoner's Dilemmas with a small subset of other agents and can form links – cooperative relationships. In contrast with most previous work on networks, in our model the links are the relationships; and the network is directed, the links' orientation capturing the presence of net gains from cooperation ("slack enforcing power") in the bilateral relation. We consider three informational assumptions: the benchmark case of complete information, where each agent observes the histories of play of all agents; the opposite case, where no information can be transmitted from an agent to the others on their observed history of play; and the case where, while agents meet to transact, they can choose to exchange and pass on received information on the respective histories of play. In this last case we assume that time is required for information to travel from one agent to the other, and allow for different speeds of information circulation within the network. We begin by characterizing sustainable networks where agents can only have relations with two neighbors. We show that when relations are asymmetric, there is a kind of an "end-network effect" (resembling the "end-game effects" of finitely repeated games), and network structures such as trees are not sustainable. Circular networks overcome this problem, ensuring that all agents' defections would be met with punishment, which provide a clear and intuitive explanation to the importance attributed by sociologists like Coleman (1988, 1990) to the "closure" of social networks.

We then show that the possibility of transmitting information about defections to other agents in the network is never used in equilibrium if enforcement relies on unrelenting "grim trigger" punishment strategies: when this is the case, once an agent deviates, a contagious process eliminates all prospect of future cooperation in the network, which removes all incentives to transmit information. With "forgiving" punishment strategies agents may instead choose

to transmit information to keep on cooperating in the rest of the network while punishing multilaterally one deviator. We also find that under imperfect information and unrelenting punishment strategies, bilaterally enforceable relations between some agents may hinder the stability of larger networks containing these agents because these may not be willing to sacrifice their relation to perform their part in the punishment phase that could sustain the larger network. This problem, though, can also be overcome with the use of relenting punishments. In contrast to results in other literatures (e.g. Kranton, 1996; Spagnolo, 2002), in our model improved outside options, like a more efficient spot market, may under certain conditions foster cooperation by making the breakup of a relation in the case of a deviation a credible threat. Extending the analysis to more complex network architectures where agents may have more than two partners/neighbors we provide formal definitions for individual and communities' "social capital" in the spirit of Coleman and Putnam that generalize the definition introduced in Spagnolo (1999a, 2000) based on Bernheim and Whinston (1990).

In an appendix we also allow agents to exclude 'cheaters' permanently from the network and then close the gap by creating new relations. We identify (quite restrictive) conditions for these strategies to form a sustainable network and show that they may marginally improve over the other strategies considered only if information about agents' history can be transmitted over the network. With full information these strategies do just as well as the others and with no information transmission they cannot sustain a network because of lack of information on whom to link to.

Related Literature. Besides being related to the already mentioned relational contracts literature, this paper contributes to the literature on the emergence and stability of networks. Prominent contributions to this literature - elegantly surveyed in Jackson (2003) - include, among others, Jackson and Wolinsky (1996), who model the emergence and stability of a social networks when agents choose to set up and maintain or destroy costly links using the notion of pairwise stability; Bala and Goyal (2000a) who consider the setup of directed and non-directed links by one agent only; Johnson and Gilles (2000), who introduce a spatial cost structure leading to equilibria of locally complete networks; Bala and Goyal (2000b), who explore the role of communication reliability in networks; and Kranton and Minehart (2000, 2001) who introduce investment and competition after in a buyer-seller network where buyers choose links in the first stage. Belleflamme and Bloch (2003) model the formation of networks of market-sharing collusive relations between firms. These models focus on agents' decision whether to build and maintain a link or not. The common central question is: Given a value of a network, a sharing rule and the cost of maintaining a link, which networks will emerge in equilibrium, and are they efficient? The underlying game and enforceability problems are left out of consideration.⁵ Our approach is complementary. We depart from this literature by explicitly modelling the underlying game, which allows us to study the consequences of its features for the stability of network structures; by focussing on the equilibrium sustainability of network structures rather

⁵In a footnote of their introduction, Belleflamme and Bloch write: "In this paper, our focus is on the stability of market sharing agreements, and we assume that these agreements are enforceable. The issue of enforceability of market sharing agreements is an important one, which cannot be answered in traditional models of repeated oligopoly interaction. We leave it for further study." Our work can be seen as a first part of this further study.

than on the process of network formation; and by showing that the condition for sustainability of each relation of which a network is composed is generally not independent of the network's architecture.

Related work that explicitly models enforcement problems in communities has mostly focused on random matching games. Kandori (1992), Ellison (1994) and others consider repeated random matching prisoner's dilemma games, showing how much cooperation can be sustained under no information transmission between agents. More recently, a similar framework is used by Dixit (2003a) to study the effects of different types of third-party enforcement, and in Dixit (2003b) to analyze the efficiency of relational vs. explicit governance systems when distance among agents differ, inviting in his conclusion to endogenize information transmission. Groh (2002) extends this approach by including an endogenous decision to pass on information to other agents, hence he is closest to our framework. In contrast to this literature, we consider situations where agents with potentially changing opponents establish long-term relationship with fixed partners (e.g. neighbors). This introduces an important forward induction element into strategic behavior when defecting. We keep Groh's endogenous choice whether to pass on information on past actions and introduce the further possibility to pass on information received by partners in the underlying game.

Our work is probably closest to the simultaneous and independent work of Haag and Lagunoff (2002) and Vega Redondo (2003).⁶ Haag and Lagunoff examine a planner's optimal choice of social linkages - or "neighborhood structure" - when each agent plays symmetric repeated prisoner's dilemma games with those other agents selected to be her neighbors, the agents' discount factors differ and are stochastically determined after the planners' choice, information is assumed to be transmitted along the links, and agents sustain cooperation by a kind of stationary grim trigger strategies. Among other things, they find a trade off between suboptimal equilibrium punishment (due to imperfect monitoring) and excessive social conflict (linked to heterogeneous discount factors). Our approach is similar in so far that we also look at the effect of different network structures on the maximum level of cooperation sustainable. However, our approaches are very different in most other respects. In their model, as in Kranton and Minehart (2000, 2001), the presence of a link is a pre-condition for interaction hence for a cooperative relation. In our model, instead, the link is the relation and there is no link without cooperation. Moreover, we allow for general asymmetries in payoffs, so that the same agent can be very interested in cooperating with one agent but ready to cheat with the other, and consider in detail the effect of different strategies besides grim trigger. Finally, we endogenize information transmission and characterize the relation with different punishment strategies.

Vega Redondo models the evolution of a social network where social relations are idiosyncratic bilateral repeated prisoners' dilemmas with symmetric payoffs, subject to random shocks.⁷ In his model, links are created and destroyed by agents depending on the expected net gains from cooperation; information is assumed to flow across the network one link per period; and enforcement power is transmitted to non sustainable relations. As in Spagnolo (1999a, 2000),

⁶We are grateful to Sanjeev Goyal who let us know about these complementary papers.

⁷See also Jackson and Watts (2002), who analyze the process of network formation when agents interact in coordination games.

"social capital" is defined as the slack enforcement power from cooperative relations that can be used to enforce cooperation in other relations where bilateral cooperation is not sustainable. Vega Redondo is mainly interested in the formation and evolution of social networks. He assumes circulation of information in the network and focuses on symmetric situations and grim trigger strategies. In contrast, we do not deal with network formation and evolution but dig more in depth in terms of sustainability of given network structures, allowing for asymmetries, different punishment strategies and agents' choice of whether to pass or conceal information. Among other things, we show that a network of relations may sustain relations none of which is sustainable if agents rely only on bilateral punishment mechanisms; and that information transmission among agents is not consistent with the use of unforgiving strategies such as "grim trigger" or "Nash-reversion".

Finally, our work is also closely related to the theoretical literature on multimarket contact and collusive behavior sparked by the seminal work of Bernheim and Whinston (1990). In their model, collusion between two firms is fostered by tying collusive behavior in one market to collusive behavior in the other thereby pooling asymmetries in incentive constraints in the two markets.⁸ The closest paper within this strand of literature is probably Maggi (1999), who adapts and extends the multimarket contact framework modelling multilateral self-enforcing international trade agreements. We generalize and extend the work of Bernheim and Whinston by considering imperfect information and endogenous information transmission, and most importantly by showing that agents/firms can easily exploit *indirect multimarket contact* to sustain otherwise unfeasible cartels where absolutely no multimarket contact is present. We generalize and extend Maggi's work by considering generic strategic situations and generic number of agents and relations, and by characterizing the role of different information transmission mechanisms and punishment strategies on networks stability.

We proceed with the definition of a network of relations in section 2. In section 3, we derive results for sustainable networks. We give an interpretation of these results to social capital in section 4. Section 5 concludes. Appendix C extends the analysis by assuming that in a punishment phase, new relations can be created.

2 The model

Interaction There is a set $N = \{1, \dots, n\}$ of infinitely lived agents $i \in N$ able to interact in pairs according to a connection structure \mathcal{C} of two element subsets of N , where $ij \in \mathcal{C}$, $i, j \in N$, if they are connected. Denote \mathcal{C}_i the set of connections of agent i . In each period t , connected agents play according to a generic prisoners' dilemma with idiosyncratic payoffs given by the

⁸Spagnolo (1999a) extends the setting to objective functions submodular in payoffs from different markets and shows that multimarket contact may facilitate collusion even in the absence of asymmetries. Matsushima (2001) introduces imperfect monitoring and shows that when firms meet in a sufficient number of markets efficient collusion can be sustained under almost the same conditions as with perfect monitoring.

following matrix:

		agent j	
		C^{ji}	D^{ji}
agent i	C^{ij}	$c^{i,j}, c^{j,i}$	$l^{i,j}, w^{j,i}$
	D^{ij}	$w^{i,j}, l^{j,i}$	$d^{i,j}, d^{j,i}$

where $l^{i,j} < d^{i,j} < c^{i,j} < w^{i,j}$ and $l^{i,j} + w^{i,j} < 2c^{i,j}$, $\forall i, j \in N$, $i \neq j$. The stage game is assumed to be constant over time. Note that the assumptions on the payoffs imply the static Nash equilibrium characterized by (D^{ij}, D^{ji}) . One interpretation of agent i 's actions C^{ij} and D^{ij} is that agent i is either taking a cooperative action C^{ij} with respect to j , or not taking it, i.e. taking no action at all, D^{ij} .

We can think of C^{ij} as "contributing" to any kind of local public good, "complying" with the terms of any relational agreement, or "colluding"; and to D^{ij} as "not...". The asymmetric prisoner's dilemma structure captures the essential strategic features of most of the examples discussed in the introduction⁹.

Agents are assumed to interact repeatedly. Time is discrete, and all agents are assumed to share a discount factor $\delta < 1$. For simplicity, we assume additive separability of agents' payoffs across interactions and across time¹⁰. Agents are assumed to choose actions which maximize their discounted utility.

Relations and relational networks In this subsection, we define what we mean by a relation and by a network of relations and give some definitions useful for analyzing these networks. We start by defining a relation:

Definition 1 (Relation) *Given a strategy profile, two agents i and j share a relation if they repeatedly play C^{ij}, C^{ji} .*

Let $R \subset \mathcal{C}$ denote the set of connections between agents who share a relation and $R_i = \{j \mid ij \in R\}$ the set of agents with whom i shares a relation.

For notational convenience, let g^{ij} denote player i 's net expected discounted gains from the relation with player j , i.e. the difference between the discounted payoff from playing (C^{ij}, C^{ji}) forever and defecting and playing the static Nash equilibrium (D^{ij}, D^{ji}) forever after

$$g^{ij} \equiv c^{i,j} - (1 - \delta)w^{i,j} - \delta d^{i,j}.$$

In a standard bilateral repeated game setting both conditions, $g^{ij} \geq 0$ and $g^{ji} \geq 0$, are necessary for a cooperative relation to be sustainable in equilibrium as, in the repeated prisoner's dilemma, Friedman's (1971) grim trigger (or "unrelenting Nash reversion") strategies are optimal in the sense of Abreu (1988). Note also that if $g^{ij} \geq 0$ player i does not have an incentive to defect from a cooperative agreement in an infinitely repeated prisoners' dilemma where players use optimal punishment strategies; but $g^{ij} < 0$ does not mean that there is no gain for agent i

⁹Matsushima (2001) shows this in detail for *quantity setting oligopolies*, where firms simultaneously choose either a small amount of supply ("cooperation") or a large amount of supply ("defection").

¹⁰Removing this (standard) assumption, along the lines of Spagnolo (1999a, 1999b), would complicate the analysis but leave all qualitative results unaffected.

from cooperation with agent j . It just means that agent i would like to deviate and bilateral cooperation is, therefore, not sustainable. We call a relation of player i with player j deficient for player i if $g^{ij} < 0$ and non-deficient for player i if $g^{ij} \geq 0$.

Definition 2 (mutual, unilateral, bilaterally deficient relation) *The relation ij is called mutual iff $g^{ij} \geq 0$ and $g^{ji} \geq 0$, it is called unilateral iff either $g^{ij} < 0$ and $g^{ji} \geq 0$ or $g^{ij} \geq 0$ and $g^{ji} < 0$, it is called bilaterally deficient iff $g^{ij} < 0$ and $g^{ji} < 0$.*

We are now in the position to define a network of relations.

Definition 3 (Relational network) *A relational network $\mathcal{N}^S = (N, R)$ is a graph consisting of the set of agents N and the set of relations R .*

Definition 4 (Sustainable relational network) *A relational network $\mathcal{N}^S = (N, R)$ is sustainable iff the strategy profile prescribing the relations in R is a sequential equilibrium.*

Definition 5 (Stable sustainable relational network) *A sustainable relational network $\mathcal{N}^S = (N, R)$ is strategically stable if it fulfills Kohlberg and Mertens' (1986) stability criteria.*

Graphical representation A simple way to represent relational networks is graphical, where a line or an arrow is drawn from agent j to agent i if $ij \in R$. This is standard in the literature. We would like to emphasize, however, that *our graphical representation of relational networks departs from the conventional graphical representation* in the networks formation literature. There, an arrow outgoing from a vertex i usually depicts a link sponsored or formed by vertex i . In our graphical representation, on the other hand, the presence of arrows conveys information on the sustainability of relations with optimal bilateral punishments, more specifically on each agent's net discounted gains from defecting from a bilateral relation: We depict a relation $ij \in R$ with $g^{ij} \geq 0$ by an *incoming arrow* to player i .

A unilateral relation, thus, is depicted by an arc originating from the agent for whom the relation is deficient. A mutual relation is depicted by an incoming arc to both players. A bilaterally deficient relation is just a line. If two agents i, j can take an action w.r.t. each other, i.e. $ij \in \mathcal{C}$, but do not share a relation, i.e. $ij \notin R$, we depict this by a dotted line. Refer to figure 1: Agents 1 and 2 share a mutual relation, the relation between 2 and 3 is unilateral – it is deficient for player 2 and non-deficient for player 3 – and agents 1 and 3 share a bilaterally deficient relation. Finally, agents 4 and 1 are connected in the sense that $14 \in \mathcal{C}$, however $14 \notin R$, i.e. $4 \in \mathcal{C}_1$ but $4 \notin R_1$.

Definition 6 (mutual, non-mutual, mixed relational network) *A relational network is mutual if it only consists of mutual relations; it is non-mutual if it does not contain mutual relations; and it is mixed if it consists of both, mutual and other relations.*

As we are going to use – to some (limited) extent – graph theoretical language, let us define the used concepts here. In the relational network, agents i and j are called adjacent from/to each other or *directly connected* if $ij \in R$. The set of agents with whom i shares relations are

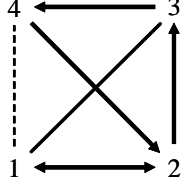


Figure 1: Graphical representation of a network of relations

the *neighborhood* of i , denoted by R_i , and $j \in R_i \Leftrightarrow i \in R_j$. Given $\mathcal{N}^S = (N, R)$, the number of agents in N is called the *order* of \mathcal{N}^S and the number of relations in R the *size* of \mathcal{N}^S . The number of arcs directed into agent i is called the *indegree* of agent i , denoted by $\text{id } i$. The *degree* of vertex i is the number of edges of agent i , denoted $\text{deg } i$. An agent of degree 1 is called end vertex. The network in figure 1 is of order 4 and size 5, there is no end vertex, and 2 is a vertex with $\text{deg } 2 = 3$ and $\text{id } 2 = 2$. A network is called an $i - j$ *path* if it consists of a finite alternating sequence of agents and links that begins with agent i and ends with agent j , in which each link in the sequence joins the agent that precedes it in the sequence to the agent that follows in the sequence, in which no agent is repeated. An $i - j$ path is called a *cycle* if $i = j$. A cycle of size c is called a c -*cycle*. A disconnecting set of edges is a set $F \subseteq R$ s.t. $\mathcal{N}^S - F$ has more than one component (connected subgraph of \mathcal{N}^S that is not contained in any other connected subgraph of \mathcal{N}^S).

Information Structures We will consider the following four information transmission mechanisms.

- (I1) *Perfect Information Transmission*: Every player observes the actions taken by any other player in the network¹¹.
- (I2) *No Information Transmission*: Every player observes only the actions taken within their own interactions. This observation cannot be transmitted.
- (I3) *Network Information Transmission (R)*: (1) Every player observes only the actions taken within their own interactions. (2) Every player has the choice whether or not to transmit this information to players he shares a relation with, $j \in R_i$. (3) Every player has the choice whether or not to transmit received information to players he shares a relation with, $j \in R_i$. A message can travel over $v > 0$ links per period.

Our information transmission mechanisms relate to the literature on perfect, public, and private monitoring in the following way. *Perfect Information Transmission* (I1) implies perfect monitoring. *No Information Transmission* (I2) implies perfect monitoring for agents i and j on their bilateral history of play, but private monitoring for the same agents on the history of play of other agents and of their neighbors with other neighbors¹². With *Network Information*

¹¹Alternatively: Every player observes only the actions taken within their own interactions. This observation is transmitted truthfully and immediately to all other players in the network.

¹²See Mailath and Morris (1999) for an example of private monitoring where the private signal about the other players' actions is imperfect.

Transmission (I3), a temporal modification of (*I2*) is assumed. In figure 2, let $v = 2$ and $t = 5$. If there is not incentive to lie for anyone in the network, player 1 perfectly monitors all actions that happened in $t = 1, \dots, 3$. Actions between agents 5 and 6 in periods 4 and 5 are assumed to be private w.r.t. agent 1. The network information transmission regime introduces, therefore, a space-time neighborhood structure into relational networks, in the sense that perfect monitoring may travel through the network with time. Note that there is no public monitoring in any of our information structures¹³.

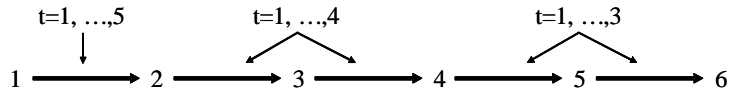


Figure 2: Agent 1's possible "observations"

Specificity We assume fully specific relations, i.e. such that if a relation between two agents breaks down, these agents cannot substitute it with relations with other agents (i.e., it is not possible for an agent to substitute a partner with another one)¹⁴. Little changes (apart from notation) if agents are assumed imperfect substitutes, in the sense that a relation with an agent can be replaced at a finite but high cost with a relation with another agent. We allow for costless substitution in appendix C, so that punishment through exclusion/replacement becomes an option, and find that the results of the present paper continue to apply: relational networks where defecting agents are excluded and the relations shared with them replaced by relations between the defecting agent's former neighbors are either not sustainable, or not strategically stable in the sense of Kohlberg and Mertens (1996).

3 Analysis

In this paper, we will be dealing with two types of multilateral strategies, multilateral grim trigger (*MG*) strategies and multilateral repentance (*MR*) strategies. For these strategies, we will give conditions for fixed relational networks to be sustainable. We will do so first for the benchmark *Perfect Information Transmission (I1)*. The results in this case resemble those from the multimarket contact literature (see Bernheim and Whinston, 1990). However, we also show that the equivalent to Bernheim and Whinston's results for different markets is not strategically stable if *MG* strategies are used and we will clarify the implication of this result for higher order networks. We will then introduce the case of *No Information Transmission (I2)*. Finally, we will cover the two different versions of *Network Information Transmission (I3)*.

Before giving results specific to the IT mechanisms defined, let us give some general first insights. Let us state without proof a necessary condition for multilateral punishment mechanisms in a relational network:

¹³For an example of public monitoring, see Green and Porter (1984). They assume that players observe their own actions, but only an imperfect public signal about the actions of the other players.

¹⁴The most obvious examples of such situations are networks where a geography limits the set of potential partners of each agent, or where agents perform different functions (e.g. they supply different goods/services).

Lemma 1 $\text{id } i \geq 1 \forall i \in R$ is a necessary condition for a relational network to be sustainable.

This is a straightforward generalization of the sustainability condition for a bilateral relational contract: For each contracting party, the net gain from cooperating has to be non-negative. The following proposition follows immediately:

Proposition 1 *End-network effect: The only sustainable non-mutual non-circular relational network is the empty one (independent of the discount factor and the information structure).*

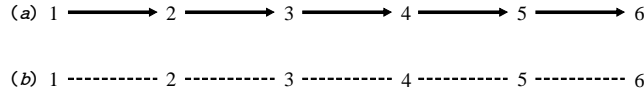


Figure 3: Only the empty network (b) is sustainable

As long as all relations are not mutual, they are not sustainable by a multilateral mechanism within a non-circular network. Figure 3 illustrates this: Part (a) shows a network that is not sustainable. In that situation, agent 1 always has an incentive to deviate and the only sustainable network is empty, as shown in (b).

Proposition 1 highlights an end-network effect much similar to the end-game effect of standard finite games and rather general. Relaxing assumption 1, it is straightforward to see that this effect generalizes to *trees* (see figure 4 for an intuition), *stars* and any other network forms where there are vertices that have only outgoing arrows.

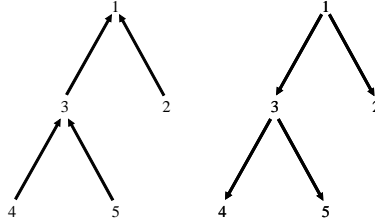


Figure 4: Trees

3.1 Perfect Information Transmission

One way to ensure that the necessary condition from lemma 1 is satisfied in a non-mutual network is to "close" the network. If agents 1 and 6 from figure 3 shared a unilateral relation that was non-deficient for 1, as in Figure 5, then each agent in the network would have an incoming and an outgoing arrow, so that a multilateral punishment mechanism may exploit payoff asymmetries.

To capture this effect, we define a network version of Friedman's (1971) "grim trigger" strategies, the *MG* strategies (*S1*) for the *Perfect Information Transmission* case (*I1*).

Strategy profile (*S1*): Every player $i \in \mathcal{N}^S$

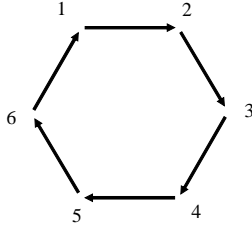


Figure 5: Circular unilateral network

1. starts playing $C^{ij} \forall j \in R_i$,
2. continues playing $C^{ij} \forall j \in R_i$ as long as he observes $C^{mn} \forall m, n \in \mathcal{N}^S$, and
3. reverts to $D^{ij} \forall j \in R_i$ for ever otherwise.

The resulting relational network is sustainable if for each player cooperating with all his neighbors is better than deviating from playing cooperative w.r.t. any subgroup of them and facing retaliation from all neighbors. If a player considers deviating from his relations with any subgroup of his neighbors, he faces immediate retaliation from all neighbors and, thus, he can just as well (and should optimally) deviate from all his relations. Proposition 2 states this general sustainability result in terms of net gains from cooperation.

Proposition 2 *Under Perfect Information Transmission (I1), a relational network is sustainable if and only if $\sum_{j \in R_i} g^{ij} \geq 0 \forall i \in \mathcal{N}^S$.*

Note that this condition encompasses also situations of partial cooperation where players choose to cooperate only in some interactions, but not in others.

Proposition 3 *If the relational network under (I1) relies on MG strategies, and it can be divided into a subnetwork that would be sustainable without the rest of the network and a subnetwork that would not, then it is not strategically stable.*

Proof. Divide network \mathcal{N}^S into subnetworks A and B such that A is sustainable by MG strategies on its own and B is not. Then there exists a player i , who is element in A and B , who has negative net gains from cooperation with his neighbors in B . By deviating only from his relation(s) in B , and then entering into the prescribed punishment with all neighbors, i would be taking an action that is strictly dominated by deviating from all his relations and then entering into the prescribed punishment with all neighbors. Ruling out the play of strictly dominated actions in the normal form of the game gives, thus, rise to a profitable deviation for this player: The other members of A must conclude that, by deviating only from his relation(s) in B , the rational player i is proposing a different - focal - equilibrium, the relational network A (which is the only reason why he would do that). ■

There are two remarks regarding the argument of proposition 3. First, it applies to Bernheim and Whinston's differences in markets result, too. There, slack enforcement power is transferred from a market in which firms can collude (e.g. because there is little competition in that

market) into one in which, absent multimarket contact, they cannot (e.g. because there is some competition in that market). Then, by *only* deviating in the more competitive market one of the firms active in the market in which they can collude would induce the same forward induction argument. Second, except if all relations are mutual or all partitions of the network are sustainable without the rest of the network, i.e. except for cases in which there is no role for the large network, sustainable higher degree networks (networks with players with more than two neighbors) contain at least one subnetwork that is sustainable without the rest of the network and one subnetwork that is not. In the consequence, the argument described in proposition 3 applies to a number of interesting cases, and multilateral agreements should contain the least necessary number of relations that make the relations sustainable.

3.2 No Information Transmission

In proposition 1, we stated that the only sustainable non-mutual non-circular relational network is the empty one. This was independent of the information structure. For Perfect Information Transmission, we showed that relational networks, including non-circular ones, are sustainable as long as for each player deviating w.r.t. all neighbors and being punished subsequently by all of them is worse than cooperating with all of them. For non-circular networks, this means that they may be sustainable if end node players have a positive net gain such as in figure 6 (a).

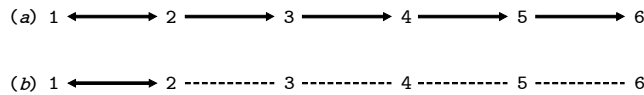


Figure 6: Non-circular relational networks that may be sustainable under (a) info structure (I1), (b) info structure (I2)

However, as this equilibrium relies on the fact that player 1 knows when player 2 cheated on player 3, which subsequently enables player 3 to rely on player 1's punishment power, this is not feasible if we are in a No Information Transmission (I2) environment.

Proposition 4 *Under information structure (I2), there is no sustainable relational network containing a disconnecting set of relations F of any one player i , for which $g^{ij} < 0 \forall ij \in F$.*

Proof. Cooperation in relations of a player i for which $g^{ij} < 0$, have to be enforced by a retaliation threat from the rest of the network. As there is no information transmission, and as F is a disconnecting set of relations of player i , actions in the other relations of i cannot be a function of the history of i 's interactions in F . Player i , thus, does not have to fear retaliation in his other relations for deviating from relations in F . ■

The result of proposition 4 can be seen in figure 6. The network in figure 6 (a) is not sustainable under (I2) as, for example, player 2 can profitably deviate from his relation with 3 without fearing retaliation from 1. The disconnecting set of relations in this case is the relation between 2 and 3.

Another example of a network that may be sustainable under (I1) but not under (I2) is that of figure 7 (a). In that network, there is a disconnecting set of relations of player 1, his

relations with 2 and 4, for which $g^{12} < 0$ and $g^{14} < 0$. The only way to discipline 1 in these relations would be by retaliation in his relation with 5, which is not feasible. It is impossible to construct a strategy that relies on punishment from a non-connected part of the network. A look at figure 7 (b) reveals that, in (I2), the end-network effect described in proposition 1 is a special case of proposition 4.

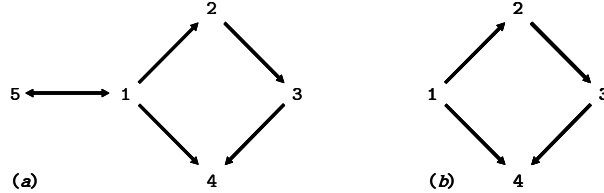


Figure 7: Relational networks that may be sustainable under (I1), but not under (I2)

Many insights can be gained by examining networks with a restricted number of neighbors. For the time being we, therefore, simplify the analysis by focussing on networks with nodes of a maximal *degree of two*, i.e. where each agent can have at most two neighbors¹⁵. We will discuss below how the results generalize to more complex networks.

Assumption 1 $\deg i \leq 2$.

Mutual relations can be sustained by direct bilateral punishments, so if all relations are mutual, a network cannot improve on what agents can sustain bilaterally. A relational network plays a role when it allows to sustain unilateral or bilaterally deficient relations, i.e. relations that would not be sustainable in the absence of a network. In this section, we explore how relational networks can be sustainable even if they do not contain any relation sustainable in the absence of such a network. We will show how the network's ability to pool payoff asymmetries and redistribute sanctioning power and information improves on what agents could achieve through bilateral interaction.

One – not sustainable – non-mutual network with a maximal degree of two, which we already examined, is the line (see figure 3). We saw that, under (I1), it is possible to have sustainability if agent 1 and agent 6 of figure 3 share a relation with $g^{1,6} \geq 0$. This insight holds for (I2) as well, using the contagious strategies: Players cooperate as long as no neighbor cheated on them and revert to the static Nash equilibrium otherwise. As we will see for mixed networks, it is necessary, though, that they hold consistent beliefs, which give them an incentive to carry on contagion in case they have a positive net gain from cooperation with a neighbor. Let us define the strategy and belief profile (S2) for the No Information Transmission (I2) environment.

Strategy and belief profile (S2): Every player $i \in \mathcal{N}^S$

1. starts playing $C^{ij} \forall j \in R_i$,

¹⁵This assumption may represent a time constraint: It is always possible not to take an action w.r.t. someone you are connected to, however, it takes time to indeed take a *cooperative* action.

2. believes that every player played C with all neighbors and goes on playing $C^{ij} \forall j \in R_i$ as long as he observes $C^{ji} \forall j \in R_i$, and
3. assigns a probability p_k to the event that player k was the first to deviate and reverts to $D^{ij} \forall j \in R_i$ for ever otherwise.

With (S2), each player in a cycle will be punished on both sides once the contagious breakdown of cooperation reaches his neighbor, and we have the following equivalent to proposition 4.

Proposition 5 *Let a non-mutual c-cycle be such that $\forall i, g^{i,i+1} < 0$ and $g^{i,i-1} \geq 0$. Under No Information Transmission (I2), that c-cycle is sustainable if and only if*

$$\forall i \in \mathcal{N}^S \delta^{c-2} g^{i,i-1} + g^{i,i+1} \geq 0.$$

Proof. Refer to figure 8 (a). *Sufficiency:* Consider (S2). The optimal deviation for an agent i in period $t = \tau$ is: play $D^{i,i+1}$ in $t = \tau$, and play $D^{i,i-1}$ in $t = \tau + c - 2$. This deviation will not be profitable if

$$\delta^{c-2} g^{i,i-1} + g^{i,i+1} \geq 0.$$

Since cooperation with $i + 1$ is upheld by the threat of retaliation from $i - 1$, after observing $D^{i-1,i}$, "infecting" is optimal. For the non-mutual c-cycle, this is true for *any* belief about the history of the game. *Necessity:* During the infinite punishment phase, agents play their minmax strategy. This is the strongest punishment available. As there is no possibility to transmit information on past behavior, it is also not possible to enter a punishment phase with $i - 1$ earlier. For these two reasons, (S2) are optimal punishments in the non-mutual c-cycle, which shows necessity. ■

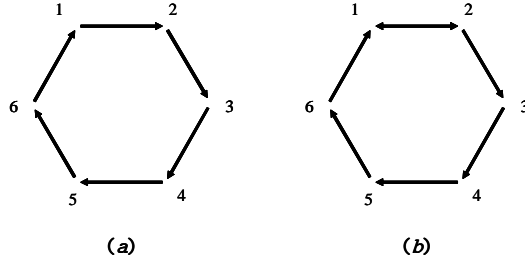


Figure 8: Non-mutual 6-cycle (a) and mixed 6-cycle (b)

Let us now give up the assumption that the network does not contain mutual relations and check under which conditions proposition 5 still holds. In figure 8 (a), consider an increase in the cooperation payoff c^{12} such that the relation 12 becomes mutual as in figure 8 (b). The contagious equilibrium given by strategies (S2) relies on the fact that each agent, who was cheated upon by a neighbor, has an incentive to start playing D with all neighbors. Assume that agent 1 observes $D^{6,1}$. For this observation, (S2) demands that he plays $D^{1,6}$ and $D^{1,2}$ forever thereafter. Is it optimal to play $D^{1,2}$ immediately after observing $D^{6,1}$? Remember that under (I2), contrary to the full information environment (I1), it is not possible to identify the

initial deviator. This means that 1 does not know whether 2 will enter into a punishment phase with him in the next period. He only has beliefs over that. As in figure 8 (b), 12 is a mutual relation, player 1 would like to play $D^{1,2}$ as late as possible and may, thus, be reluctant to enter into an punishment phase with 2 immediately, if he believes that 2 enters the punishment phase with a low probability. This leads to proposition 6.

Proposition 6 *Let a sustainable non-mutual c-cycle be such that $\forall i, g^{i,i+1} < 0$ and $g^{i,i-1} \geq 0$. For agent k increase $c^{k,k+1}$ such that $g^{k,k+1} \geq 0$, so that the network becomes mixed. Then, for sufficiently high $l^{k,k+1}$ and sufficiently low $w^{k,k+1}$, (S2) does only result in a sustainable network if beliefs are such that k assigns a sufficiently high probability to the event that $k+1$ or a player close to $k+1$ was the first to deviate.*

Proof. With the beliefs specified in (S2), if agent 1 in figure observes $D^{6,1}$ and $C^{2,1}$ in $t = \tau$, he assigns probability p_j to the event that player j deviated first. Then with probability $1 - p_2 - p_3$, player 1 does not expect player 2 to play $D^{2,1}$ in $t = \tau + 1$ or $t = \tau + 2$. Then to wait one more period before playing $D^{1,2}$ is not profitable if $p_2 + p_3$ is large, or if $l^{1,2}$ is sufficiently small or $w^{1,2}$ is sufficiently high. ■

Proposition 6 shows a trade-off between profitability and sustainability of cooperation in networks: A player, who benefits (too) strongly from relations with everybody he is connected to, may hurt cooperation between other players because he may be unwilling to enter a punishment phase.

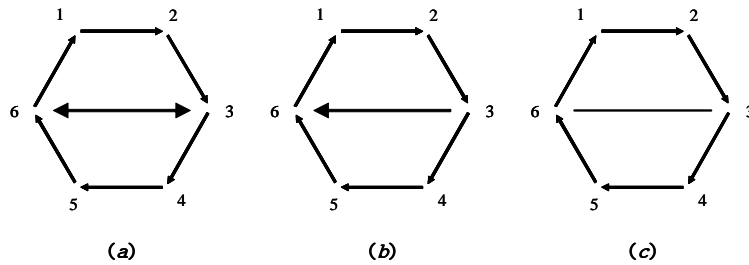


Figure 9: Adding a relation to a circular, non-mutual network

Let us now relax the assumption $\deg i \leq 2$. In particular, let us add a link to an existing cycle and study the conditions for which the resulting network is sustainable. Refer to figure 9. The sustainability conditions from proposition 5 generalize to

$$\begin{aligned} g^{6,1} + \delta^2 g^{6,3} + \delta^4 g^{6,5} &\geq 0, \\ g^{3,4} + \delta^2 g^{3,6} + \delta^4 g^{3,2} &\geq 0, \text{ and} \\ g^{i,i+1} + \delta^2 g^{i,i-1} &\geq 0 \end{aligned}$$

for $i = 1, 2, 4, 5$. Furthermore, generalizing proposition 6, if any of the subnetworks is sustainable without the rest of the network, then the beliefs have to be such that in case of a deviation of a neighbor enough probability weight is put on the event that a neighbor from the sustainable subnetwork was the one who deviated first.

Relaxing the assumption $\deg i \leq 2$, it is also straightforward to see that figure 6 generalizes to lines that are adjacent to subnetworks which are sustainable in autarky. Assume in figure 10 that the subnetwork $(\{1, 2, 3\}, \{12, 23, 31\})$ is sustainable in autarky, i.e. without making use of possible relations 34, 45, 56. Then, under (I1), strategies (S1) make (a) a sustainable network if, in addition, $g^{31} + g^{32} + g^{34} \geq 0$, $g^{43} + g^{45} \geq 0$, and $g^{54} + g^{56} \geq 0$, whereas network (b) is the only sustainable one under (I2), irrespective of the payoffs in the relations 34, 45, and 56.

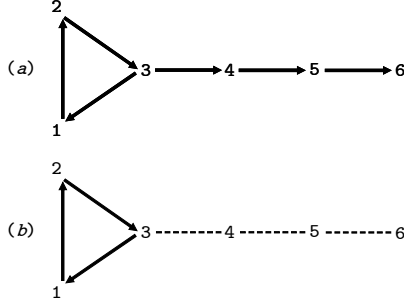


Figure 10: Non-circular network with a (possibly) sustainable subnetwork at one end.

Remark. *Statements made on mutual relations apply to subnetworks that are sustainable in the absence of the rest of the network.*

3.3 Network Information Transmission

Assumption 2 *If they are indifferent between lying and telling the truth, players tell the truth.*

Let us now turn our attention to the Word-of-Mouth communication. In this environment, players have the choice whether or not to transmit (truthfully or not) information about their history of play and about messages they received whenever they meet to cooperate. As players do have the possibility to lie, there is an equivalent for proposition 4 under (I3).

Proposition 7 *Under information structure (I3), there is no sustainable relational network containing a disconnecting set of relations F of any one player i , for which $g^{ij} < 0 \forall ij \in F$.*

Proof. Cooperation in relations of a player i for which $g^{ij} < 0$, have to be enforced by a retaliation threat from the rest of the network. As player i has the possibility to lie, and as F is a disconnecting set of relations of player i , actions in the other relations of i cannot be a function of the history of i 's interactions in F . Player i , thus, does not have to fear retaliation in his other relations for deviating from relations in F . ■

To continue, let us – once more – first restrict our attention to networks with $\deg i \leq 2$.

Proposition 8 *Let a non-mutual c -cycle be such that $\forall i, g^{i,i+1} < 0$ and $g^{i,i-1} \geq 0$. If punishment of a deviation of a player requires infinite Nash reversion of in his interactions, then under the network information transmission regime (I3), the network is sustainable if and only if $\forall i \in \mathcal{N}^S \delta^{c-2} g^{i,i-1} + g^{i,i+1} \geq 0$, regardless of the speed of information transmission.*

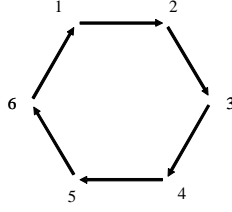


Figure 11: A 6-cycle

Proof. Assume the network information transmission regime (*I3*) and unforgiving strategies. Suppose that player i observes a deviation of his neighbor $i - 1$. Then, since, due to the unforgiving strategies, there will never be a return to cooperation with $i - 1$, the best response of i in his (i 's) remaining deficient relation would be to deviate from that relation. As information can only be transferred during cooperation, no use is made of the potential high speed of information transmission, leaving only room for the same strategies as under (*I2*). ■

As we see from proposition 8, an important feature of our model is that the design of the punishment paths interacts with agents' incentives to transmit information. There are two important implications of that feature.

One implication is that, even though grim trigger strategies are optimal punishment strategies in all the bilateral relations (i.e. if they rely on bilateral punishment mechanisms), for non-mutual relational networks, the *MG* strategies (*S1*) and (*S2*) are *optimal* punishment strategies only for the *complete information* (*I1*) and the *no information transmission* case (*I2*), respectively. They are optimal because punishment is as strong as possible on both sides, once it arrives there, *and* it arrives on both sides with the smallest possible delay. Under the *network information transmission* regime (*I3*) with high speeds of information transmission instead, i.e. in a world where information can be transmitted via links and this information travels more than one link per period, strategies (*S2*) we will show them *not* to be *optimal* anymore. The potentially high speed of information transmission is – individually optimally – not being used, and therefore, punishment "on the other side" arrives later than necessary, reducing the enforcement power of the network. We will below introduce multilateral repentance strategies, which use information transmission and we will show that they entail optimal punishment.

A second implication is that, with *MG* strategies, also for (*I3*), the sustainability of the relational network is sensitive to the underlying belief structure. Word-of-Mouth communication (or Network Information Transmission) is assumed to happen when players meet to cooperate, as a by-product of the meetings. As a consequence, in case of defection, no information on the identity of the first cheater can be transmitted, and players need to rely on beliefs, as in (*I2*). We, thus, state without proof the (*I3*) equivalent to proposition 6.

Proposition 9 *Let a sustainable non-mutual c -cycle be such that $\forall i, g^{i,i+1} < 0$ and $g^{i,i-1} \geq 0$. For agent k increase $c^{k,k+1}$ such that $g^{k,k+1} \geq 0$, so that the network becomes mixed. If punishment of a deviation of a player requires infinite Nash reversion of in his interactions, then under the network information transmission regime (*I3*), for sufficiently high $l^{k,k+1}$ and sufficiently low $w^{k,k+1}$, the network is only sustainable if beliefs are such that k assigns a sufficiently high*

probability to the event that $k + 1$ or a player close to $k + 1$ was the first to deviate.

Similarly, the results we found for (I2) when relaxing the assumption that $\deg i \leq 2$ hold again, if in (I3) players use *MG* strategies. Refer again to figure 9. The sustainability conditions from proposition 8 again generalize to

$$\begin{aligned} g^{6,1} + \delta^2 g^{6,3} + \delta^4 g^{6,5} &\geq 0, \\ g^{3,4} + \delta^2 g^{3,6} + \delta^4 g^{3,2} &\geq 0, \text{ and} \\ g^{i,i+1} + \delta^2 g^{i,i-1} &\geq 0 \end{aligned}$$

for $i = 1, 2, 4, 5$. Furthermore, generalizing proposition 9, if any of the subnetworks is sustainable without the rest of the network, then the beliefs have to be such that in case of a deviation of a neighbor enough probability weight is put on the event that a neighbor from the sustainable subnetwork was the one who deviated first. Furthermore, once more relaxing the assumption $\deg i \leq 2$, it is also straightforward to see that figure 6 generalizes once more to lines that are adjacent to subnetworks which are sustainable in autarky, such as in figure 10.

With *MG* strategies, the potential of high speed information transmission is therefore left unused. Transmitting information cannot be an equilibrium choice with because the punishment phase lasts forever. A defection leads then to a complete breakdown of the relational network during the punishment phase¹⁶, and agents prefer to "grab what they can" before the collapse of the network by defecting/infecting rather than maintaining the relation and transmitting information. By rewarding agents for transmitting information instead of infecting their neighbor, it becomes possible to avoid the breakdown of cooperation and to make use of high speeds of information transmission, thereby, relaxing the agents' incentive constraint and allowing a sustainable network for a lower δ than (S2). Proposition 10 shows this. For that end, we define multilateral repentance strategies (S3) in appendix A, which require cheaters to "pay back" all their neighbors by cooperating while being cheated on for some time period, and players who have been cheated upon to go on cooperating with all their other neighbors and to send a message about the cheating along the network. Neighbors of cheaters gain from punishing, information is transmitted, and punishment can be as hard as possible (equivalent to minmax forever) as fast as possible.

Proposition 10 *Let \mathcal{N}^S be a c -cycle. Let $\underline{\delta} \equiv \{ \delta | g^{i,i+1} + \delta^{c-2} g^{i,i-1} = 0 \}$. Let $\tilde{\delta}$ be the minimum discount factor for which, together with appropriate $T_j, \forall j \in \mathcal{N}^S - (S3)$ constitutes a sustainable non-mutual network. Then*

- (i) $\tilde{\delta} \leq \underline{\delta}$ with a strict inequality for high speeds of information transmission (for $v > 1$).
- (ii) if one substitutes non-mutual subnetworks with mutual ones the network is still sustainable $\forall \tilde{\delta}$.

For the proof, which we relegate to appendix , there are four incentive constraints to consider:

¹⁶That holds also if one considers a change in (S2) such that the reversion to the stage Nash equilibrium does not last forever but only for T periods.

1. Every agent has to have an incentive to stick to $C^{ij} \forall j \in R_i$ as long as neither he observes D^{ji} for a $j \in R_i$ nor he receives a message containing D^{jn} for $j \in R_i$. (IC^{CI})
2. Given one neighbor j of i played $D^{j,i}$, each agent m (including $m = i$ herself) has to have an incentive to send a message containing $D^{j,i}$ to her other neighbor n and stick to $C^{m,n}$. (IC^{CII})
3. Every neighbor of an original cheater has to have an incentive to carry out the punishment. (IC^P)
4. Every original cheater has to agree to be punished. (IC^{LP})

We first show that (IC^{CII}) and (IC^P) are never binding. Using (IC^{LP}) and (IC^{CI}), we then show that, for a speed of $v = 1$, it is possible to choose a length $T_j, \forall j \in \mathcal{N}^S$, of the punishment period for each agent such that the punishment payoff for her is equivalent to minmaxing her on both sides forever¹⁷, i.e. the strength of the punishment is equivalent to the one for ($S2$). Increasing the speed of information transmission reduces the delay of the punishment and, thus, relaxes (IC^{LP}) which in turn gives room to make it more severe. This establishes (*i*). Since agents are being rewarded for punishing their neighbor, they always have an incentive to do so during a punishment phase even if they want to cooperate bilaterally, which establishes (*ii*).

Corollary 1 *Under network information transmission ($I3$), for high enough ν it is possible to find a $T_j \forall j \in \mathcal{N}^S$ such that (**S3**) is an optimal punishment mechanism whereas ($S2$) is not.*

Proof. Two elements determine the strength of the multilateral punishment mechanism in the network: the payoff after punishment starts on each side, and the promptness with which this punishment starts on each side after a deviation. It is always possible to adjust the length of the punishment phase T_j for each player j such that he receives an punishment payoff equivalent to minimax forever. Furthermore, the other neighbor of cheater can "get to know" about the defection and start the punishment phase at the earliest with the information that travelled through the network. This means that ($S3$) is an optimal punishment mechanism. As for high ν , information transmission is faster than contagion, ($S2$) is not an optimal punishment mechanism for high ν . ■

Punishment with ($S3$) is as strong as possible and as fast as possible, therefore these are the optimal (punishment) strategies in our network. Proposition 10 also shows that it is not necessary to have a complete breakdown of cooperation in the network in case of a deviation if information about past actions can be transmitted. The equilibrium is, thus, more robust (against e.g. mistakes) and increases welfare during punishment phases.

Remark 1 *Since under perfect information ($I1$) the agent that defects first is known, the complete breakdown of the network in a punishment phase can be avoided through punishments as in*

¹⁷To avoid divisibility problems, one can always assume a public randomization device giving the end of the punishment period for each agent such that in expectation the punishment payoff of the initial deviator is equivalent to minimaxing him forever.

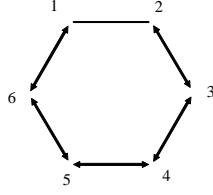


Figure 12: Mixed relational network containing only mutual relations except one bilaterally deficient one

(S3). These strategies¹⁸ result in the same critical discount factor as for (S1), as punishment was immediate on both sides already with (S1). While strategy profile (S3) avoids the breakdown of the network due to mutual subnetworks for (I3), it can not be used under (I2) since it makes use of information transmission. Without information transmission it is impossible to know who deviated first from the equilibrium path and a targeted punishment of only the agent that defects first becomes unattainable.

Remark 2 Up to now, we have not explicitly considered bilaterally deficient relations. It should however be clear at this point that a mixed circular relational network containing bilaterally deficient relations – as for example the network in figure 12 – is sustainable with the same strategies discussed above under the same conditions given.

4 Social Capital

In this section we show that there are generalizations of the results we obtained for the simple relational networks above, allowing for more than two neighbors¹⁹. For this end, we will use a c -cycle as a basic structure and add a link. We will show how networks of relations that generate “slack enforcement power” for some agents may enable these to sustain cooperation on additional deficient relations, even in *one shot* prisoner’s dilemma interactions. We then offer an interpretation of this use of networks of relations as cooperation-enforcement/governance devices for new social dilemmas in terms of the highly debated but somewhat vague concept of “social capital”.

Social capital à la Coleman Consider figure 13. We have seen that agents 1’s and 6’s membership in the network may help them sustain a bilaterally deficient relation between them. This is the case if the sum of the net gains from cooperating for i and k from their other relations are large enough, i.e. if they dispose of sufficient *slack enforcement power* to enforce the additional relation.

¹⁸All neighbors $j \in N_i$ of an initial cheater i start playing $D^{j,i}$ until i has played $C^{i,j} \forall j \in N_i$ for T periods and then they go back to playing $C^{i,j}, C^{j,i}$. In all other games in the network, the players go on playing the cooperative action during the punishment phase for player i . As the initial cheater can always get his minimax payoff forever, which is the payoff from the punishment in (S1), the biggest T , for which this strategy profile is an equilibrium, gives him exactly this payoff.

¹⁹We have done so already in the sections before when we looked at trees, stars, or non-circular networks, one end node of which was an autarkically sustainable subnetwork.

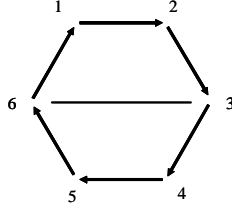


Figure 13: Social Capital: Sustaining a one-shot interaction with the threat of retaliation in the social network

Suppose the circular network $\{12, 23, 34, 45, 56, 61\}$ is a social network, i.e. the relations in it are *social relations*, and suppose the bilaterally deficient relation between i and k is a *one-shot prisoner's dilemma*, say an occasional business transaction where each agent can "hold up" the other. Then the slack enforcement power from our social network, used to govern a one-shot business interaction, is much like what Coleman (1990) defines as *social capital*:

Social capital is defined by its function. It is not a single entity, but a variety of different entities having two characteristics in common: They all consist of some aspect of social structures, and they facilitate certain actions of individuals who are within that structure. Like other forms of capital, social capital is productive, making possible the achievement of certain ends that would not be attainable in its absence. Like physical capital and human capital, social capital is not completely fungible, but is fungible with respect to certain activities. A given form of social capital that is valuable in facilitating certain actions may be useless or even harmful for others. Unlike other forms of capital, *social capital inheres the structure of relations between persons and among persons*. It is lodged neither in individuals nor in physical implements of production. [Italic our]

This is a micro-perspective on social capital. Our model allows for a formal definition for social capital à la Coleman:

Definition 7 (*Social capital à la Coleman*): In a sustainable social network \mathcal{N}^S with $i, k \in \mathcal{N}^S$, we define the **individual social capital** i and k can draw upon to govern a one-shot Prisoner's Dilemma interaction ik as

$$sc_{ik} = \left(\max \left\{ w^{ik} - c^{ik}, w^{ki} - c^{ki} \right\} \middle| C^{ik}, C^{ki} \text{ is equilibrium in a MPM containing } \mathcal{N}^S \text{ and } ik \right).$$

The social capital agent i can draw on from being part of a social network is defined as the slack enforcement power usable to enforce cooperation-compliance in other interactions in need of governance through an *MPM* (multilateral punishment mechanism)²⁰. With complete

²⁰This is a definition of social capital that quantifies it by its returns. A similar approach has been taken by Fernandez et al. (2000), who measure the returns to bonuses paid to current employees who successfully refer new employees out of their personal network. Fernandez et al. (2000) refer to the bonus paid to the employees as the company's "social capital investment" and to the net cost savings they obtain in their recruitment process as "the return to social capital". In our opinion, these references are, however, misleading. What the authors call

information (I1), this is only a player specific definition as it is equivalent to the sum of his net gains from cooperation in all his social relations $sc_{ik} = \min \left\{ \sum_{j \in R_i} g^{ij}, \sum_{j \in R_i} g^{kj} \right\}$. For the other information regimes, the extent to which existing relations in a social network can facilitate "the achievement of certain ends" for an agent depends not only on his net gains from cooperation, i.e. how much he has to lose in his social relations. Since the delay with which an eventual punishment sets in matters, it also depends on *partners' locations* in the network.

Example 1 *To give an example for circular networks (or subnetworks), one could think of firms located on a (Salop) circle, with different capacities in the left and right market, cooperating/colluding with their neighbors. Coleman (1990) insists on the importance of the "closure" (circularity) of social networks. Giving a graphical representation as in figure 14²¹, he suggests that if parents (A and B), whose children (a and b) are friends, share a relation, too, as in figure 14 part (a), they have more "power" over their children – thanks to what Coleman calls "intergenerational closure" – than if they do not, as in figure 14 part (b). Lack of relations among parents makes it more difficult for them to successfully impose/enforce norms on/upon their children. He does not provide a game theoretical foundation for his claim, but our model fits precisely his story.*

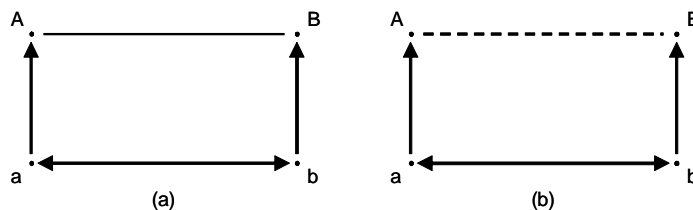


Figure 14: Representation of two communities: (a) with and (b) without intergenerational closure (adapted from Coleman, 1990).

Reviewing "(game-)theoretical questions stimulated by a reflection on social capital", Sobel (2002) identifies two ways in which Coleman's (1990) network closure or – put differently – "dense social networks make enforcement of group cooperative behavior more effective": First by creating "common knowledge of information", and second by increasing "the quality and reliability of third-party monitoring needed to enforce cooperative dynamic equilibria." With this paper, we offer an additional explanation of why closure might be important for the enforcement of cooperative behavior, the pooling of payoff asymmetries.

Social capital à la Putnam Robert Putnam (1995) takes another perspective on social capital. For him, the concept "refers to the collective value of all 'social networks' and the

"investment in social capital" should be termed more appropriately *the employees' return to their social capital*: The investment has been done by the employees themselves, and is not quantified by the firm's referral bonus. Furthermore, the net cost savings of the firm in the hiring process, which the authors label as "the returns to social capital", is more appropriately *the firm's return to the bonus paid to its employees*. It is part of, but not the whole, return to the investment in social capital. It reflects the relative bargaining power between employees and the firm in sharing the return to social capital.

²¹Note that his representation differs from ours by using two arrows to describe one relation.

inclinations that arise from these networks to do things for each other.” This is a macro-perspective on social capital, which, translated into our model, lead to the following formal definition:

Definition 8 (*Social capital à la Putnam*): In a sustainable social network \mathcal{N}^S with $i, k \in \mathcal{N}^S$, we can define the **social capital of a society** as the average individual social capital in that society

$$\frac{1}{n \text{ card}(R_i)} \sum_{i \in \mathcal{N}^S} \sum_{k \in R_i} sc_{ik}.$$

The conclusion to be drawn from our model for the construction of aggregate measures of social capital is: If there is full information about the actions of economic agents, it suffices to have a measure of the average sum of the net gains from cooperation per person from social relations in the economy. However, if this is not the case, as in most real world situations, in addition, a measure of the density of the network should be used.

Information transmission as social capital The value of the social network rests of course also in its ability to enforce transmission of information on the history of interactions between network members and "outsiders". If outsiders interact repeatedly with changing members of the network, transmission of information on the history of these interactions through the network helps governing such transactions by deterring outsiders' opportunism. In this sense, our model can also be seen as a microfoundation of Kandori's (1992) *attaching a label to a cheater* by the members of the social network, and of Sobel's second reason why dense networks help enforce cooperative behavior – increasing "the quality and reliability of third-party monitoring needed to enforce cooperative dynamic equilibria".²²

5 Conclusion

Each of us is involved in networks of long term relationships of different kinds and with different parties. In our model, agents maintain long term self-enforcing relations thanks to the information circulation and the enforcement/sanctioning power ensured by the network(s) such relations form. We identify equilibrium conditions for different network architectures, paying special attention to differences between circular and non-circular ones. The basic framework is that of repeated games between fixed partners with three basic information structures: complete information, private monitoring with no information transmission, and private monitoring with information transmission through the network's links.

We show that the pooling of asymmetries in payoffs across the network may allow agents to sustain relations that would not be sustainable otherwise under all three informational assumptions. We find an end-network effect, i.e. that a non-circular non-mutual network is not sustainable, and that the possibility to transmit information about defection through the links is

²²Acemoglu and Zilibotti (1999) stress the value of information transmission mechanisms for a society at the macro-growth level.

not exploited in equilibrium if enforcement relies on unforgiving punishment phases. More complex punishment strategies induce agents to use information transmission and keep cooperating in the rest of the network while punishing a defection (which increases efficiency and decreases the discount factor necessary to sustain the network). If information can be transmitted via the network, grim trigger strategies, therefore, cease to be optimal punishments, as they do not use the possibility to transmit information to punish cheaters faster. Having self-sustaining relations in the network turns out to hurt cooperation with imperfect information, because agents may then not be willing to perform the prescribed punishment after a defection. When information can be transmitted, the network may be sustained using strategies that reward the punisher and encourage information transmission.

The model provides a microfoundation to Granovetter's (1985) idea of "embeddedness", and the "end-network effect" provides a clear explanation of why "closure" of social networks is so important for social capital, as argued by Coleman (1988) and (1990). It also allows us to derive formal measures of social capital, both at individual and aggregate level. Immediate applications include the organization of inter-firm relations in industrial districts, the enforcement of collusive behavior in business networks, interbank relations and the effects of "social capital" on the governance of economic and social interactions. In her much acclaimed book, Saxenian (1994) attributes a large part of Silicon Valley's success to a special culture of cooperation in that industrial district stemming from a common background of the early workforce in that area. We believe our model offers a complementary explanation how social networks may facilitate information circulation in a community.

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APPENDIX

A Strategy profile (S3)

Strategy profile (S3)

1. Each player i start by playing $C^{ij} \forall j \in R_i$.
2. Each player i go on playing $C^{ij} \forall j \in R_i$, as long as he does not observe D^{ji} for any $j \in R_i$, and as long as he does not receive a message containing D^{jn} for some $j \in R_i$.
3. If player i observes D^{ji} for any $j \in R_i$ and she received no message about an earlier defection of j , he
 - (a) sends a message about the deviation to her other neighbor and goes on playing C with him,
 - (b) plays D^{ij} until j and i played (D^{ij}, C^{ji}) for T_j periods,
 - (c) sends her other neighbor a message about the end of the punishment phase for player j , and
 - (d) goes back to 2. thereafter.
4. If a neighbor k of j receives a message about j 's initial deviation, he
 - (a) plays D^{kj} until both, she receives the message that D^{ij}, C^{ji} has been played for T_j periods and D^{kj}, C^{jk} has been played for T_j periods, and
 - (b) returns to 2. thereafter.
5. If player j played D^{ji} , he
 - (a) plays C^{ji} for the next T_j periods, D^{jk} in the period when k receives the information on her initial deviation and C^{jk} for the next T_j periods, and
 - (b) returns to 2. thereafter.
6. Each player truthfully passes on the messages.
7. If some player deviates from the actions in 3. – 5., the punishment starts against this player.

B Proposition 10

For notational convenience the following definition will be useful.

Definition 9 *We define a function*

$$\theta(c, v) \equiv \begin{cases} \max\left\{\frac{c-2}{v}, 1\right\} & \text{if } \text{int}\left(\frac{c-2}{v}\right) = \frac{c-2}{v} \\ \max\left\{\text{int}\left(\frac{c-2}{v}\right) + 1, 1\right\} & \text{if } \text{int}\left(\frac{c-2}{v}\right) \neq \frac{c-2}{v} \end{cases} .$$

This function maps the order of the cycle c and the speed of information transmission v into the strictly positive natural numbers and indicates the period in which an information about play between agents i and $i + 1$ in period 0 reaches agent $i - 1$.

In the proof we first consider the incentive constraints for agents in the network not to deviate from cooperation in phase I (IC^{CI}), from cooperation with their other neighbor in phase II that is if one neighbor cheated (IC^{CII}), from punishing the original cheater in phase II (IC^P), and from letting the others punish when she deviated in the first place (IC^{LP}). In a second step we show that $\tilde{\delta} \leq \underline{\delta}$. It is shown that IC^{CII} and IC^P are never binding, so we can concentrate on IC^{CI} and IC^{LP} . For a speed of $v = 1$, by an appropriate choice of the length of the punishment, the conditions for cooperation can be made equivalent to the ones for (S2). Increasing the speed then relaxes IC^{LP} which gives room to make punishment more severe, which establishes (i): $\tilde{\delta} \leq \underline{\delta}$. Since agents are being rewarded for punishing their neighbor, they always have an incentive to do so during a punishment phase even if they want to cooperate bilaterally, which establishes (ii). If T is chosen such that punishment is as hard as playing minimax strategies with both neighbors forever, this is the hardest punishment possible. Since here information transmission is used, every mean to decrease the delay before punishment on both sides sets in is used. This establishes the corollary.

Proof. The following incentive constraints are to be satisfied:

1. (IC^{CI}) For each agent i , playing $D^{i,i+1}$ in $t = 0$ and $D^{i,i-1}$ in $t = \theta(c, v)$, which is her best deviation, yields $w^{i,i+1}$ in $t = 0$, $l^{i,i+1}$ for the following T_i periods and $c^{i,i+1}$ thereafter, as well as $c^{i,i-1}$ until $t = \theta(c, v) - 1$, $w^{i,i-1}$ in $t = \theta(c, v)$, $l^{i,i-1}$ for the following T_i periods and $c^{i,i-1}$ thereafter. Playing $C^{i,i+1}$ and $C^{i,i-1}$ forever yields $\frac{1}{1-\delta} (c^{i,i+1} + c^{i,i-1})$. Summing up leads to (IC^{CI}), which is the condition for (S3) to be a Nash equilibrium.

$$\begin{aligned}
IC^{CI} \equiv & (c^{i,i+1} - w^{i,i+1}) + \sum_{t=1}^{T_i} \delta^t (c^{i,i+1} - l^{i,i+1}) \\
& + \delta^{\theta(c,v)} (c^{i,i-1} - w^{i,i-1}) + \sum_{t=\theta(c,v)+1}^{\theta(c,v)+T_i} \delta^t (c^{i,i-1} - l^{i,i-1}) \geq 0 \\
& \forall i \in N^S, i+1, i-1 \in R_i.
\end{aligned}$$

2. (IC^{CII}) Suppose that in period $t = 0$, agent $i - 1$ played $D^{i-1,i}$.

- (a) Suppose $\theta(c, v) \geq T_{i-1} - 1$. Then nothing changes in the trade-off in his interactions with $i + 1$ from IC^{CI} . In his interactions with $i - 1$, i will already have returned to the cooperative phase, which means he will give up $c^{i,i-1}$ for T_i periods by infecting $i + 1$. Thus, i is in the same situation as if he never had been cheated on by $i - 1$, which means $IC^{CII} = IC^{CI}$.

$$IC^{CII} = IC^{CI} \quad \text{if } \theta(c, v) \geq T_{i-1} - 1,$$

- (b) Suppose now $\theta(c, v) < T_{i-1} - 1$. Again nothing changes in the trade-off in his interactions with $i + 1$ from IC^{CI} . Thus the first line of IC^{CII} coincides with the first line in IC^{CI} . If in $t = 1$, agent i plays $D^{i,i+1}$ instead of sticking to cooperation and just sending a message, this results in agent $i + 1$ sending a message that reaches agent

$i - 1$ in $t = \theta(c, v) + 1$. This yields agent i a utility of $l^{i,i-1}$ until $t = \theta(c, v) + T_i + 2$. By sticking to cooperation, she would have had a utility of $w^{i,i-1}$ from $t = \theta(c, v) + 1$ until $t = T_{i-1}$ and of $c^{i,i-1}$ from $t = T_{i-1} + 1$. This difference constitutes the second and third line of IC^{CII} .

$$\begin{aligned}
IC^{CII} &\equiv (c^{i,i+1} - w^{i,i+1}) + \sum_{t=1}^{T_i} \delta^t (c^{i,i+1} - l^{i,i+1}) \\
&\quad + \sum_{t=\theta(c,\nu)+1}^{T_{i-1}-1} \delta^t (w^{i,i-1} - l^{i,i-1}) + \sum_{t=T_{i-1}}^{\theta(c,\nu)+T_i} \delta^t (c^{i,i-1} - l^{i,i-1}) \geq 0 \\
&\quad \forall i \in \mathcal{N}^S, i+1, i-1 \in R_i \quad \text{if } \theta(c, v) < T_{i-1} - 1,
\end{aligned}$$

Since

$$IC^{CI} - IC^{CII} = \begin{cases} \sum_{t=\theta(c,\nu)}^{T_{i-1}-1} \delta^t (c^{i,i-1} - w^{i,i-1}) < 0 & \forall \theta(c, v) < T_{i-1} - 1 \\ 0 & \forall \theta(c, v) \geq T_{i-1} - 1 \end{cases},$$

whenever IC^{CI} holds, IC^{CII} is satisfied.

3. (IC^P) Suppose agent i receives the message that agent $i + 1$ deviated in their relation with one of their other neighbors. Then agent i has to have an incentive to punish him. Since $w^{i,j} > c^{i,j}$ together with (IC^{CI}), this is always the case.
4. (IC^{LP}) Suppose in period $t = 0$, agent i played $D^{i,i+1}$. Then he has to agree to playing $(C^{i,i+1}, D^{i+1,i})$ for T_i periods instead of his minimax strategy forever. After having played $D^{i,i+1}$ in $t = 0$, for agent i sticking to punishment strategies means incurring $l^{i,i+1}$ for T_i periods and $c^{i,i+1}$ thereafter. It furthermore means $w^{i,i-1}$ in $t = \theta(c, v)$, $l^{i,i-1}$ for the following T_i periods and $c^{i,i-1}$ thereafter. Deviating from punishment strategies yields $d^{i,i+1}$ forever, $w^{i,i-1}$ in $t = \theta(c, v)$ and $d^{i,i-1}$ forever thereafter. The difference between these utilities is represented by (IC^{LP}).

$$\begin{aligned}
IC^{LP} &\equiv \sum_{t=0}^{T_i-1} \delta^t (l^{i,i+1} - d^{i,i+1}) + \sum_{t=T_i}^{\infty} \delta^t (c^{i,i+1} - d^{i,i+1}) \\
&\quad + \sum_{t=\theta(c,\nu)}^{\theta(c,\nu)+T_i} \delta^t (l^{i,i-1} - d^{i,i-1}) + \sum_{t=\theta(c,\nu)+T_i+1}^{\infty} \delta^t (c^{i,i-1} - d^{i,i-1}) \geq 0 \\
&\quad \forall i \in \mathcal{N}^S, i+1, i-1 \in R_i.
\end{aligned}$$

Constraint (IC^{CI}) consists of addends that are either strictly increasing in δ or strictly positive. Constraint (IC^{LP}) is strictly increasing in δ for $\delta \in (0, 1)$. Both conditions do not hold for a δ close to 0. They do hold strictly for a δ close enough to 1, thus there exists a $\tilde{\delta}$ for which both constraints hold. Therefore under the conditions stated, strategy (S3) is subgame perfect for $\delta > \tilde{\delta}$.

Since $l^{i,j} < d^{i,j}$, it is possible to fix a $T_i \forall i$ such that $IC^{LP} = 0$ ²³. Given that T_i , assume $v = 1$, such that $\theta(c, v) = c - 2$. For this, IC^{CI} is satisfied for all δ that satisfy $\delta^{c-2}g^{i,i-1} + g^{i,i+1} \geq 0$. Now consider $v > 1$. Again, it is possible to fix a $T_i \forall i$ such that $IC^{LP} = 0$. That ensures the same strength of the punishment. But now the punishment in the non-deficient relation sets in earlier which reduces the value of the deviation and therefore for $v > 1$, $\tilde{\delta} < \underline{\delta}$.

Since agents are being rewarded for punishing their neighbor, they always have an incentive to do so during a punishment phase even if they want to cooperate bilaterally, which establishes (ii).

If T_i is chosen for each agent i such that the punishment is as hard as playing minimax strategies with both neighbors forever, this is the hardest punishment possible. Since here information transmission is used, every mean to decrease the delay before punishment on both sides sets in is used. This establishes the corollary. *Q.E.D.* ■

C Sustainable networks with creation of new links

In this appendix, we consider the permanent exclusion of an agent from the relational network together with the assumption that the remaining members close the gap in it by establishing a new relation.

As before, we examine exclusion for the three information transmission regimes. We will define exclusion, and then show that only for the network information transmission (I3), exclusion equilibria may sustain networks with lower discount factors than the mechanisms examined before. This is true, however, only under quite restrictive conditions.

Definition 10 (Punishment by exclusion) *We define punishment by exclusion as the permanent choice of a cheater's neighbors in the relational network to play the non-cooperative action w.r.t. the cheater and their permanent choice to play the cooperative action w.r.t. each other.*

As in section 3, to begin with, we assume $\deg(i) \leq 2$. Contrary to section 3, however, we assume that every two agents of the relational network are able to interact with each other:

Assumption 3 $\exists ij \in \mathcal{C} \forall i, j \in \mathcal{N}^S, i \neq j$.

We assume that it is impossible to find an agent outside the network in order to substitute for an agent in the network. One could think of a specific group of agents exchanging a kind of service for which it is impossible to find providers outside the community:

Assumption 4 *If $\nexists ij \in R$ for some j then $\nexists ik \in \mathcal{C}$ for any $k \in \mathcal{N}^S$.*

As in section 3, we want to explore the possibility to sustain relations through the network which would otherwise be non-sustainable, i.e. we focus our attention at non-mutual networks. In addition, we assume that agents can have at most one relation with each other, i.e. are not able to exploit direct multimarket contact:

²³That means that the punishment is as strong as if the deviator was punished with infinite reversion to the static Nash equilibrium.

Assumption 5 $\forall i, j \exists$ one and only one $ij \in \mathcal{C}$.

The way we defined punishment by exclusion, defections are deterred by the creation of a new relational network. The consequence is that, if this new relational network is not sustainable, there is no deterrence. Therefore, also a deviation from it – if the same punishment is applied – has to be deterred by the existence of yet another sustainable relational network. Strategies will, thus, feature a recursive element. Given this recursive nature of exclusion and given assumption 5, strategies have to include at some point in time other punishments as well. We therefore define strategies (S4) and (S5) for the (I1) and (I2) information transmission structures, respectively, such that the punishment depends on the size of the remaining network, making the assumption that the punishment changes to defection with all neighbors if the residual network is triangular.

Strategy profile (S4)

1. Players $k \in \mathcal{N}^S$ start by playing $C^{kj} \forall k \in \mathcal{N}^S, \forall j \in R_k$.
2. Each player k goes on playing $C^{kj} \forall j \in R_k$ as long as no deviation by any player in the network is observed.
3. If an agent i played $D^{i,j}$,
 - (a) her neighbors $j \in R_i = \{i + 1, i - 1\}$ will play $D^{j,i}$ forever
 - (b) if $size(\mathcal{N}^S) > 3$, $\mathcal{N}_{-i}^S \equiv \mathcal{N}^S - \{i, i - 1; i, i + 1\} + \{i - 1, i + 1\}$, her neighbors $j \in \{i + 1, i - 1\}$ will form a link $i - 1, i + 1$ and all agents $k \in \mathcal{N}_{-i}^S$ go to point 1.
 - (c) if $size(\mathcal{N}^S) = 3$, every agent k reverts to $D^{kj} \forall j \in R_k$ forever.

Strategy profile (S5)

1. Every agent $k \in \mathcal{N}^S$ starts with $C^{k,j}$ with all neighbors k and transmits info on history as well as received info.
2. ...goes on with $C^{i,j}$ as long as he observes $C^{k,j}$ and he does not observe $C^{i,k}$ with i being not a neighbor
3. If $size(\mathcal{N}_{-j}^S) \geq 3$, and
 - (a) if k observes $D^{j,k}$ without having played $D^{k,\cdot}$ before, he will
 - i. play $D^{k,j}$ forever
 - ii. play $C^{k,i}$ w.r.t. j 's other neighbor i
 - (b) if an agent i not being a neighbor of k observes $C^{k,i}$, he will
 - i. play $D^{i,j}$ with $j \in R_k$ forever
 - ii. play $C^{i,k}$ starting from the next period
 - (c) all agents $k \in \mathcal{N}_{-j}^S$ go to point 1

4. If $size(\mathcal{N}^S) = 3$ and if k observes $D^{j,k}$, or if k played $D^{k,j}$ before, he plays $D^{k,j} \forall j \in R_k$ forever after.

Note again that a closure of the relational network – after excluding a defector – by agents who formerly did not share a relation requires that there are $ij \in \mathcal{C}$ with $ij \notin R$. Furthermore, there will be additional conditions to fulfill for these strategies to be an equilibrium.

Proposition 11 *Suppose assumptions 1 and 3–5.*

1. *Assume the perfect information transmission regime (I1). Let $\widehat{\delta} \equiv \{\delta \mid g^{i,i-1} + g^{i,i+1} = 0\}$. Let $\widehat{\delta}$ be the minimum discount factor necessary to sustain a network with (S4) under (I1). Then*
 - (a) $\widehat{\delta} = \widehat{\delta}$ *provided that all potential relations between network member that are not currently links in the network are mutual for $\widehat{\delta}$.*
 - (b) *the network is not strategically stable.*
2. *Assume the no information transmission regime (I2). Then there is no relational network sustained by exclusion.*
3. *Assume the network information transmission regime (I3). Let $\widetilde{\Delta}$ be the set of δ for which – together with an appropriate $T_j \forall j$ – (S3) constitutes a sustainable non-mutual network with $g^{i,i+1} < 0$ and $g^{i,i-1} \geq 0$ under (I3) and $\widetilde{\delta} = \min \left\{ \widetilde{\Delta} \right\}$. Let $\widetilde{\Delta}$ be the set of δ for which (S5) constitutes a sustainable non-mutual network with $g^{i,i+1} < 0$ and $g^{i,i-1} \geq 0$ under (I3) and $\widetilde{\delta} = \min \left\{ \widetilde{\Delta} \right\}$. Then*
 - (a) *if $l^{i,i-2}$ is not too small, if ν is not too high, and if all potential relations between members of the network, which are not links in the network, are mutual for $\widetilde{\delta}$, $\widetilde{\delta} < \widehat{\delta}$.*
 - (b) *the network is not strategically stable.*
 - (c) *the strategy profile (S5) is the optimal cooperative strategy profile in the class of strategy profiles with exclusion.*

Proof. *Part 1.:* We first give the conditions for sustainability of the network assuming optimal deviations given the punishment.

- i. For any $i \in \mathcal{N}^S$ we must have

$$g^{i,i+1} + g^{i,i-1} \geq 0.$$

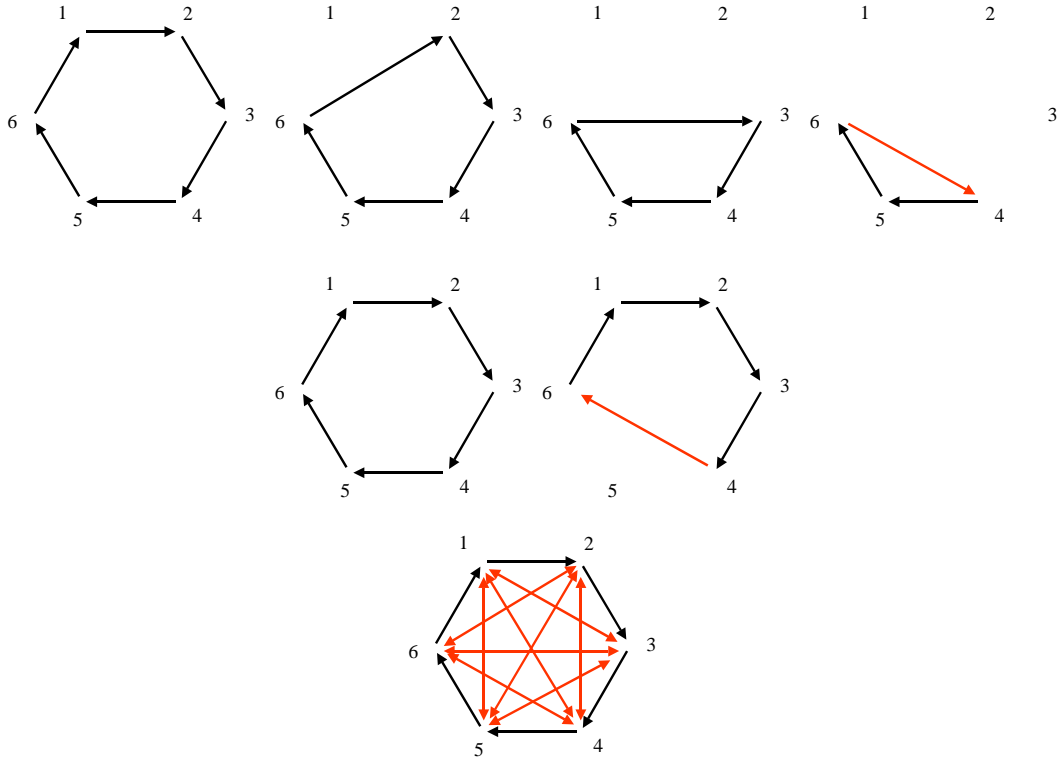
- ii. For $i - 1$ and $i + 1$, we need

$$\begin{aligned} g^{i-1,i+1} + g^{i-1,i-2} &\geq 0 \\ g^{i+1,i+2} + g^{i+1,i-1} &\geq 0. \end{aligned}$$

- iii. Points i. and ii. must hold for any member of any network \mathcal{N}_{-i}^S and any member of any reduced network thereof except the triangular networks. In the triangular one, only i. has to hold.

Part 1. (a): (S1) punishes a deviation immediately with the strongest possible punishment, i.e. the one that gives the cheater his minimax payoff forever. It is, therefore, not possible to decrease the delay until punishment takes place and the strength of the punishment.

Condition ii. together with iii. imply that not only $g^{i+1,i-1} \geq 0$, but also $g^{i+2,i-1} \geq 0$, $g^{i+3,i-1} \geq 0$, and so on. To see this, consider the following figures. The first row represents the consequences of the deterrence of a deviation of agent 1, then 2, and then 3. We see, that the relation of 4 with 6 has to be non-deficient for 4. The second row represents the consequences of a deterrence of a deviation of agent 4. We see now that the relation of 4 with 6 has to be non-deficient for 6. The consequence is that all agents inside the circle have to potentially have mutual relations. Thus, if they are for $\hat{\delta}$, then $\hat{\delta} = \hat{\delta}$.



Part 1 (b): If all $ij \notin R$ have to be mutual, this has consequences for the strategic stability of the equilibrium as shown in proposition 6.

Part 2: In order to be able to link to the neighbor of the neighbor who cheated on a player, this player has to know who is the neighbor of that cheater. This requires information on the history of his neighbor's play, which he does not have under (I2).

Part 3: Again, we first give the conditions for sustainability of the network. As before, we assume optimal deviations given the punishment.

i. No agent has to have an incentive to deviate from the cooperative action:

$$g^{i,i+1} + \delta g^{i,i-1} \geq 0.$$

ii. An agent i who has been cheated on by an agent $i - 1$ has to have an incentive to play $C^{i,i+1}$ and $C^{i,i-2}$:

$$g^{i,i+1} + (1 - \delta) l^{i,i-2} + \delta c^{i,i-2} - d^{i,i-2} \geq 0,$$

iii. and to go on playing that in the next period:

$$g^{i,i+1} + \delta g^{i,i-2} \geq 0.$$

iv. Any agent $i - 2$ who observes $C^{i,i-2}$ from a member of the network who is not his neighbor has to have an incentive to play $C^{i-2,i}$:

$$g^{i-2,i} + \delta g^{i-2,i-3} \geq 0.$$

v. Points i. through iv. must hold for any member of \mathcal{N}^S and of any network \mathcal{N}_{-i}^S and any member of any reduced network thereof except the triangular networks. In the triangular ones, only i. has to hold.

Part 3. (a): In this equilibrium, permanent Nash reversion of $i + 1$ arrives immediately. Permanent Nash reversion of $i - 1$ arrives after one period. With strategy profile (S3), a punishment of $i + 1$ as strong as permanent Nash reversion arrives immediately. The punishment of $i + 1$ as strong as permanent Nash reversion arrives after $\theta(c, \nu)$ periods. Since $\theta(c, \nu)$ is decreasing in ν , for low ν condition i. is less strict than the equivalent condition for (S3).

Conditions iii. through v. imply, similarly to conditions ii. and iii. from Part 1. of this proposition, that all potential relations between members of the network, which are not links in the network, have to be *mutual*.

Condition ii. is only less stringent than condition i. if $l^{i,i-2}$ is not too low.

Part 3 (b): As conditions iii. through v. imply that all potential relations between members of the network, which are not links in the network, have to be *mutual*, it is possible to deviate suboptimally in a network \mathcal{N}_{-i}^S which makes a punishment of an agent in that mutual relation by the other agent in that mutual relation a dominated action.

Part 3 (c): As the creation of the links between neighbors is immediate, punishment sets in as soon as possible. This punishment involves minimax strategies forever and is thus as hard as possible. *Q.E.D.* ■

Let us briefly comment on these results. First, with exclusion punishments, it is only possible to improve over the strategies defined before for the Network Information Transmission Case (I3). For (I1), these strategies do just as well, and for (I2), there is no punishment by exclusion for lack of information whom to link to. Second, as a deviation of any member of the network has to be deterred by a sustainable other network, this results in certain conditions on

the interaction structure. These *certain conditions* are quite restrictive: all potential relations between members of the network, which are not links in the network, have to be mutual. This causes the network to be not strategically stable, and thus (S4) and (S5), respectively, unlikely to be chosen as equilibrium strategies. In addition, for the Network Information Transmission Regime (I3) the loss from playing *C* if your partner plays *D* has to be not too low, and the speed of information transmission has to be not too high. This condition on l' would be relaxed if one allowed for the potential relations, which would have to be mutual, to exist in the first place, i.e. if one was to give up the "time constraint" $\text{deg}(i) \leq 2$. Third, with (S4), as with restitution punishments, agents enjoy the advantage of avoiding the breakdown of the network during a punishment phase. Thus, there is a utility gain compared with (S1). However, compared to restitution punishments (similar to (S3)), the neighbors of a cheater lose utility – a payback of the damages is not done. Furthermore, if the network fulfills any other function, such that the size of the network matters for overall welfare, there is a loss in welfare compared to restitution punishments due to the exclusion of the cheater.