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Bank Mergers, Competition, and Liquidity

We model the impact of bank mergers on loan competition, reserve holdings, and aggregate liquidity. A merger changes the distribution of liquidity shocks and creates an internal money market, leading to financial cost efficiencies and more precise estimates of liquidity needs. The merged banks may increase their reserve holdings through an internalization effect or decrease them because of a diversification effect. The merger also affects loan market competition, which in turn modifies the distribution of bank sizes and aggregate liquidity needs. Mergers among large banks tend to increase aggregate liquidity needs and thus the public provision of liquidity through monetary operations of the central bank.

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THE LAST DECADE HAS WITNESSED an intense process of consolidation in the financial sectors of many industrial countries. This “merger movement,”

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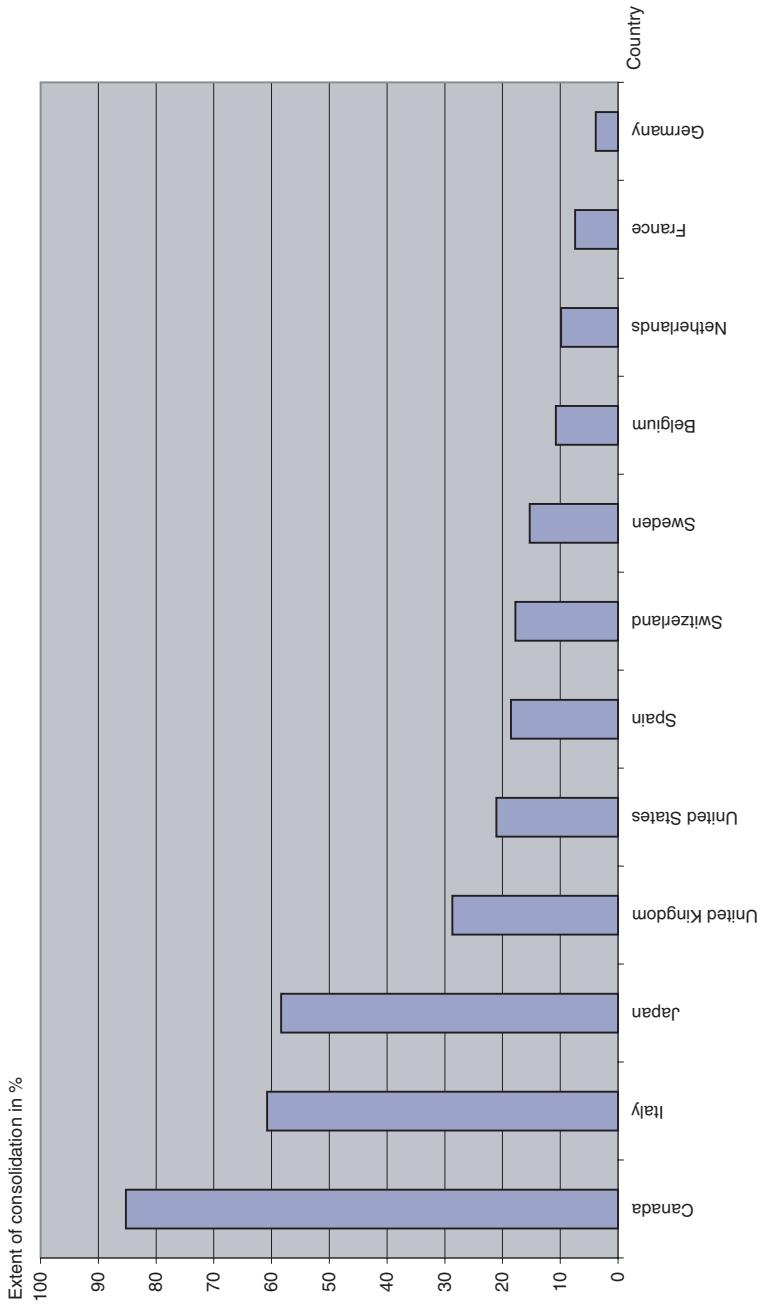


FIG. 1. Banking Consolidation in Industrial Countries, 1990-99.

NOTE: The extent of consolidation is defined as the number of domestic M&As between banks (1990-99) divided by the average number of banks (1990-99) times 100. Australia has been excluded for data consistency.
 SOURCE: Group of Ten (2001).

documented in a number of papers and official reports, was particularly concentrated among banking firms and occurred mostly within national borders.¹ As shown in Figure 1, in countries like Canada, Italy, and Japan more than half of the banks combined forces over the 1990s.

As a consequence, many countries (e.g., Belgium, Canada, France, the Netherlands, and Sweden) reached a situation of high banking concentration or faced a further deterioration of an already concentrated sector. As can be seen from Table 1, a few large banks often constitute more than 70% of the national banking sector.

This extensive consolidation process raises a number of important questions, including the effects on a nation's financial stability. The conventional wisdom is that consolidation may lower liquidity needs and reduce activity in the interbank market. For example, according to the G-10 Report on Financial Sector Consolidation,

By internalizing what had previously been interbank transactions, consolidation could reduce the liquidity of the market for central bank reserves, making it less efficient in reallocating balances across institutions and increasing market volatility. (Group of Ten 2001, p. 20)²

However, this statement may not survive in a moderately general model. Merging banks may either increase or decrease their demand for reserve assets. Moreover, mergers affect loan markets as well as deposit markets, and loan market competition also affects the demand for reserves.

To better understand the effects of bank consolidation, we develop a model that allows us to investigate the joint impact of mergers on credit market competition, banks' demand for reserves, and the functioning of the interbank market. Banks raise deposits at date 0, and invest in long-term loans to entrepreneurs and liquid short-term reserves. On the loan market banks compete in prices and retain some market power through differentiation. Reserves are needed to cope with the uncertainty about depositors' time of withdrawals. As in Klein (1971) and Diamond and Dybvig (1983), deposits are stochastic as a fraction of them is withdrawn prematurely at date 1. If deposit withdrawals (also, demand for liquidity) exceed a bank's reserve holdings, the bank incurs a cost to obtain from the interbank market the liquidity needed to satisfy depositors. Thus, a bank's demand for reserves depends on its uncertainty about deposit withdrawals and the relative cost of refinancing, that is, the ratio of the cost of borrowing on the interbank market in case of liquidity shortage to the cost of raising

1. See, for example, Boyd and Graham (1996), Berger, Demsetz, and Strahan (1999), Hanweck and Shull (1999), Dermine (2000), European Central Bank (2000), Organisation for Economic Co-operation and Development (2000), and Group of Ten (2001).

2. The effects of consolidation on interbank market liquidity are of course most pronounced in smaller countries with national money markets, such as Denmark, Sweden, or Switzerland. For example, the Swiss banking system is now dominated by two main players. In order to moderate adverse effects on liquidity, the Swiss National Bank considerably facilitated foreign banks' access to the Swiss franc money market. "With this opening the influence of the main banks on the conditions in the money market was reduced. Their share of total outstanding liquidity transactions declined from more than 80% to now around 50%" (quote from the SNB Board Member Bruno Gehrig at the Jahresend-Mediengespräch December 8, 2000, see http://www.snb.ch/d/aktuelles/referate/ref_001208_bge.html; translation by the authors).

TABLE 1
BANK CONCENTRATION RATIOS IN INDUSTRIAL COUNTRIES, 1980–98

	1980	1990	1998	Change 1980–90	Change 1990–98
<i>Europe</i>					
Belgium	53.4	48.0	66.7	−5.4	18.7
France	n.a.	51.9	70.2	n.a.	18.3
Germany	n.a.	17.1	18.8	n.a.	1.7
Italy	n.a.	(25.9)	38.3	n.a.	12.4
Netherlands	n.a.	73.7	81.7	n.a.	8.0
Spain	38.1	38.3	(47.2)	0.2	8.9
Sweden	n.a.	62.0	84.0	n.a.	22.0
Switzerland	n.a.	53.2	(57.8)	n.a.	4.6
United Kingdom	n.a.	43.5	35.2	n.a.	−8.3
<i>North America</i>					
Canada	n.a.	60.2	77.7	n.a.	17.5
United States	14.2	11.3	26.2	−2.9	14.0
<i>Pacific Rim</i>					
Australia	76.5	72.1	73.9	−4.4	1.8
Japan	28.5	31.8	30.9	3.3	−0.9

NOTES: Concentration ratios are defined as the share of the five largest banks in total bank deposits (in percent). Values in parentheses are for 1992 (Italy) or 1997 (Spain, Switzerland). Changes are in percentage points (Spain and Switzerland 1990–97, Italy 1992–98). n.a. = not available.

SOURCE: Group of Ten (2001).

more deposits and keeping more reserves initially. The interbank market redistributes reserves from banks with excess reserves to banks with shortages. However, when there is aggregate excess demand on the interbank market, the central bank must intervene to provide the missing liquidity and smooth out fluctuations in the banking system. Aggregate liquidity supply and central bank intervention can be thought of in terms of private versus public liquidity, in the spirit of Holmström and Tirole (1998). In this sense, the risk of aggregate illiquidity and the expected liquidity needs represent the intensity with which central banks monitor and intervene in the interbank market.

A merger affects banks' behavior with respect to both reserve management and loan market competition. As regards the former, the merger modifies the uncertainty about deposit withdrawals, and creates an internal money market where the merged banks can reshuffle reserves. Thus, besides the typical *diversification effect* related to the pooling of idiosyncratic liquidity shocks, the merger induces an *internalization effect*, which increases *ceteris paribus* the marginal value of each unit of reserves that can now be used to meet withdrawals at any of the two banks. The demand for reserves of the merged banks balances these two effects. We find that the internalization effect is stronger when the relative cost of refinancing is low, while the diversification effect dominates and banks reduce reserve holdings when the relative cost of refinancing is high. The intuition behind this result hinges on the relationship between the marginal value of one unit of reserves, the initial value of reserves and the precision with which banks can estimate the probability of needing liquidity at date 1. The merger changes the distribution of shocks the merged banks face and makes them less uncertain about their future liquidity needs. As a consequence, when the relative cost of refinancing is low and banks keep a low level of reserves,

the merged banks increase their reserves as they are more certain to need them at date 1. The opposite happens in case of high relative cost of refinancing. In all circumstances, however, the merged banks improve their liquidity situation, having lower liquidity risk and expected liquidity needs. Moreover, by lowering refinancing costs, the internal money market generates endogenous financial cost efficiencies, which reduce, *ceteris paribus*, the anti-competitive effects of mergers between banks. These results suggest that merged banks benefit from scope economies in their liquidity management by raising deposits in two imperfectly correlated deposit markets. This last result finds empirical support in Hughes, Land, Mester and Moon (1996), who show that banks active in imperfectly correlated deposit markets have lower costs of controlling liquidity risk, especially after consolidation.

Mergers affect market power and therefore change both loan rates and market shares in our imperfectly competitive loan market. As known from the industrial organization literature, the overall effect of a merger on loan rates depends on how strong the increase in market power is relative to potential efficiency gains. Loan rates increase when the *market power effect* dominates, and they decrease when the *cost efficiency effect* prevails. The novelty here is that the merger may generate efficiency gains through the re-optimization of their reserve holdings as well as through a potential reduction in lending costs.

The changes the merger induces in banks' reserve holdings, loan competition, and balance sheets affect also the interbank market and aggregate liquidity. We can disentangle again two channels. The first one, which we denote as *reserve channel*, originates directly from the changes induced in merged banks' reserve holdings as described above. A merger leads to higher aggregate liquidity supply and thus lower expected aggregate liquidity needs when banks increase their reserve holdings, whereas the opposite holds when banks' reserves are reduced. The second channel, the so-called *asymmetry channel*, relates instead to the distribution of balance sheet sizes across banks. A merger inducing greater asymmetry among banks increases the variance of aggregate liquidity demand, thus increasing *ceteris paribus* expected aggregate liquidity needs. In contrast, mergers inducing smaller asymmetry reduce both the variance of aggregate liquidity demand and expected aggregate liquidity needs. The impact of consolidation on aggregate liquidity depends on the interaction between the reserve and the asymmetry channel. In particular, whether the two effects work in the same or opposite directions depends on the size of the relative cost of refinancing and on how mergers affect the asymmetry of banks' balance sheets.

The model delivers several insights, which can be interpreted according to size of merger and type of country or financial system. First, mergers between large banks leading to a "polarization" of the banking system with large and small institutions are more likely to lead to higher aggregate liquidity needs than mergers involving small banks, since they increase the asymmetry in banks' balance sheets. This result is particularly noteworthy in light of Table 1, which suggests that the banking sector consolidation of the 1990s may have led to greater asymmetry in the size of banks in most industrialized countries. In particular, Belgium, Canada, France, the Netherlands, and Sweden, the five top players, enlarged their market shares significantly to very high

levels. Second, mergers are more likely, *ceteris paribus*, to increase aggregate liquidity needs in developing countries than in industrial ones, since they induce lower individual reserve holdings in less efficient markets, where banks face higher refinancing costs. Third, the effects of consolidation on loan competition and aggregate liquidity tend to be complementary in industrial countries but not in developing ones. In fact, whereas mergers are likely to affect competition and liquidity in the same direction when the cost of refinancing is low (i.e., mergers between large banks are likely to increase both loan rates and expected aggregate liquidity needs, and vice versa for mergers involving small banks), they always push toward larger expected liquidity needs when the cost of refinancing is high, independent of the effect on loan competition. Finally, the impact of bank mergers on reserve holdings and aggregate liquidity may depend on the phase of the business cycle. Mergers happening in upturns may affect reserves and private aggregate liquidity more negatively than mergers happening in downturns, at least in the short run.

Relation with the literature. Our approach to study the joint implications of bank mergers for competition, individual and aggregate liquidity combines elements of the industrial organization literature on the implications of exogenous mergers under imperfect competition with the financial intermediation literature characterizing banks as liquidity providers. As in Deneckere and Davidson (1985) and Perry and Porter (1985), banks have incentives to merge to acquire market power. Unlike these papers, however, in our model banks' incentives to merge are also driven by financing cost advantages related to size, and in particular, by the gains from the optimal adjustment of reserve holdings due to the presence of an internal money market. In this sense, our article also links the industrial organization literature on mergers with the contributions of Yanelle (1989, 1997) and Winton (1995, 1997) on the relation between competition and diversification in finite economies.

The field of research studying the role of banks as liquidity providers started with Diamond and Dybvig (1983). More recently Kashyap, Rajan, and Stein (2002) describe the links between banks' liquidity provision to depositors and their liquidity provision to borrowers through credit lines, and Diamond (1997) discusses the relationship between the activities of Diamond- and Dybvig-type banks and liquidity of financial markets. Concerning liquidity provision by public authorities, Holmström and Tirole (1998) analyze the role of government debt management in meeting the liquidity needs of the productive sector. However, this literature has not considered one of our main concerns here: the implications of imperfect competition and financial consolidation for private and public provision of liquidity.

Several authors have studied the rationale for an interbank market and its effect on reserve holdings. For example, Bhattacharya and Gale (1987) show that banks can optimally cope with liquidity shocks by borrowing and lending reserves, but they also argue that moral hazard and adverse selection lead to under-investment in reserves. Bhattacharya and Fulghieri (1994) add that with some changed assumptions reserve holdings can also become excessive. These authors argue that the central bank has a role in healing these imperfections. Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000) analyze how small unexpected liquidity shocks can lead to

liquidity shortages in the banking system and thus, in the absence of a central bank, to contagious crises. We discuss how the likelihood and the extent of such shortages vary with changes in market structure when a central bank stands ready to offset private market liquidity fluctuations through monetary operations.

The article is also related to the literature on internal capital markets. Gertner, Scharfstein, and Stein (1994) and Stein (1997) discuss the efficiency-enhancing role of these internal markets. While Scharfstein and Stein (2000) and Rajan, Servaes, and Zingales (2000) warn that they might also become inefficient if internal incentive problems and power struggles lead to excessive cross-divisional subsidies; the empirical results of Graham, Lemmon, and Wolf (2002) suggest that “value destruction” in firms is not related to consolidation, supporting the idea of efficiently functioning internal capital markets. Concerning banks, Houston, James, and Marcus (1997) provide evidence that loan growth at subsidiaries of U.S. bank holding companies (BHCs) is more sensitive to the holding company’s cash flow than to the subsidiaries’ own cash flow; and Campello (2002) shows that the funding of loans by small affiliates of U.S. BHCs is less sensitive to affiliate-level cash flows than independent banks of comparable size. Focusing on short-term assets, we show how the creation of an internal money market can cushion external liquidity shocks and how it affects banks’ reserve choices and banking system liquidity. We also show that the financing cost advantages associated with the internal money market lead the merged banks, *ceteris paribus*, to be more aggressive on the loan market.

The remainder of the article is structured as follows. Section 1 sets up the model. Section 2 derives the equilibrium before a merger (“status quo”). The subsequent section characterizes the effects of a merger on individual banks’ behavior, and Section 4 looks at its implications for aggregate liquidity. Section 5 contains a discussion of the different scenarios for competition and liquidity effects of bank consolidation. Section 6 concludes. All proofs are in the Appendix.

1. THE MODEL

Consider a three-date ($T = 0, 1, 2$) economy with three classes of risk-neutral agents: N banks ($N > 3$), numerous entrepreneurs, and numerous individuals. At date 0 banks raise funds from individuals in the form of retail deposits, and invest the proceeds in loans to entrepreneurs and in liquid short-term assets denoted as reserves. Thus, the balance sheet for each bank i is

$$L_i + R_i = D_i, \tag{1}$$

where L_i denotes loans, R_i reserves, and D_i deposits.

1.1 Competition in the Loan Market

Banks offer differentiated loans and compete on price. The differentiation of loans may emerge from long-term lending relationships (see, e.g., Sharpe 1990, Rajan

1992), specialization in certain types of lending (e.g., to small/large firms or to different sectors), or in certain geographical areas. Following Shubik and Levitan (1980), we assume that each bank i faces a linear demand for loans given by

$$L_i = l - \gamma \left(r_i^L - \frac{1}{N} \sum_{j=1}^N r_j^L \right), \quad (2)$$

where r_i^L and r_j^L are the loan rates charged by banks i and j (with $j = 1, \dots, i, \dots, N$), and the parameter $\gamma \geq 0$ represents the degree of substitutability of loans. The larger γ the more substitutable are the loans. Note that expression (2) implies a constant aggregate demand for loans $\sum_{i=1}^N L_i = Nl$, as in Salop (1979).

Processing loans involves a per unit lending cost c , which can be thought of as a setup cost or a monitoring cost. Loans mature at date 2 and yield nothing if liquidated before maturity.

1.2 Deposits, Individual Liquidity Shocks, and Reserve Holdings

Banks raise deposits in N distinct “regions.” A region can be interpreted as a geographical area, a specific segment of the population, or an industry sector in which a bank specializes for its deposit business. There is a large number of potential depositors in every region, each endowed with one unit of funds at date 0. Depositors are offered demandable contracts, which pay just the initial investment in case of withdrawal at date 1 and a (net) rate r^D at date 2. The deposit rate r^D can be thought of as the reservation value of depositors (the return of another investment opportunity), or, alternatively, as the equilibrium rate in a competition game between banks and other deposit-taking financial institutions.

As in Klein (1971) and Diamond and Dybvig (1983), deposits are subject to liquidity shocks. A fraction δ_i of depositors at each bank develops a preference for early consumption, and withdraws at date 1. The remaining $1 - \delta_i$ depositors value consumption only at date 2, and leave their funds at the bank until then.³ The fraction δ_i is assumed to be stochastic. Specifically, δ_i is uniformly distributed between 0 and 1, and it is i.i.d. across banks.⁴ This introduces uncertainty at the level of each individual bank and in the aggregate. All uncertainty is resolved at date 1, when liquidity shocks materialize.

Given the structure of liquidity shocks, each bank faces a demand for liquidity $x_i = \delta_i D_i$ at date 1 and uses its reserves R_i to satisfy it. Reserves represent a storage technology that transfers the value of investment from one period to the next. We may think of cash, reserve holdings at the central bank, or even short-term government

3. The fraction δ_i can also be interpreted as a regional macro shock. For example, weather conditions may change the general consumption needs in a region, so that each depositor withdraws a fraction δ_i of his initial investment.

4. We assume for simplicity that liquidity shocks are independent across banks, but all our results remain valid as long as liquidity shocks are not perfectly correlated.

securities and other safe and low yielding assets. (The interest rate on reserves need not be zero.) The stochastic nature of δ_i means that the realized demand for liquidity x_i may exceed or fall short of R_i , thus introducing the need for a market where liquidity can be traded at date 1, as described more below. Denoting as $f(x_i)$ the density function of x_i , at date 0 each bank faces a *liquidity risk*—the probability to experience a liquidity shortage at date 1—given by

$$\phi_i = \text{prob}(x_i > R_i) = \int_{R_i}^{D_i} f(x_i) dx_i, \quad (3)$$

and has an *expected liquidity need*—the expected size of liquidity shortage that needs to be refinanced at date 1—equal to

$$\omega_i = \int_{R_i}^{D_i} (x_i - R_i) f(x_i) dx_i. \quad (4)$$

1.3 Interbank Refinancing and Aggregate Liquidity

At date 1 an interbank market opens where banks can either borrow or lend depending on whether they have shortages ($x_i < R_i$) or excesses ($x_i > R_i$) of reserves. We focus on the ultra-short interbank or money market, such as the unsecured market for wholesale deposits, where both banks and the central bank operate.⁵ Since in this market rates are always in between the policy rates at which sound individual banks may receive (give) overnight deposits from (to) the central bank (e.g., the marginal lending and the deposit rates in the euro area, and the rate on primary credit in the United States; see, e.g., Hartmann, Manna, and Manzanares 2001, European Central Bank 2004), we assume that banks can borrow at a rate r^{LB} and lend at a rate r^{LL} , independently of the counterparty. Our focus is on the amount of public liquidity the banking system may need.

Given the presence of aggregate uncertainty, there may be an aggregate shortage or an aggregate excess of private liquidity on the market. An aggregate shortage of private liquidity occurs whenever banks' aggregate demand for liquidity is higher than the aggregate supply of private liquidity represented by the sum of individual banks' reserves, that is, whenever

$$\sum_{i=1}^N x_i > \sum_{i=1}^N R_i. \quad (5)$$

5. The most relevant and largest ultra-short market is the overnight market, in which banks exchange liquidity at the so-called “overnight” or “Fed funds” rates (e.g., bid and ask rates). Most central banks stabilize those market rates around an “official rate” (e.g., the Fed Fund target rate in the United States, and the minimum bid rate in the euro area) by adjusting the supply of liquidity to changes in the aggregate demand. Recent evidence indicates that central banks control overnight rates quite successfully (e.g., Carpenter and Demiralp 2006, Pérez Quirós and Rodríguez Mendizábal 2007).

T = 0	T = 1	T = 2
Price competition on the loan market, choice of reserves, deposits are raised	Shocks δ_i materialize, interbank market opens	Loans mature, claims are settled, and profits materialize

FIG. 2. Timing of the Model.

Denoting as $X_i = \sum_{i=1}^N x_i = \sum_{i=1}^N \delta_i D_i$ banks' aggregate demand for liquidity with density function $f(X_i)$, we express the probability with which an aggregate shortage of private liquidity occurs through the *aggregate (or systemic) liquidity risk* as

$$\Phi = \text{prob} \left(X_i > \sum_{i=1}^N R_i \right) = \int_{\sum R_i}^{\sum D_i} f(X_i) dX_i, \tag{6}$$

and its expected size through the *expected aggregate (or systemic) liquidity needs* as

$$\Omega = \int_{\sum R_i}^{\sum D_i} \left(X_i - \sum_{i=1}^N R_i \right) f(X_i) dX_i. \tag{7}$$

The aggregate liquidity risk (6) and the expected aggregate liquidity needs (7) can then be interpreted as measures of the degree to which the banking system depends on the public supply of liquidity, in the spirit of Holmström and Tirole (1998). Formulated differently, they are indicators of the frequency and the size of central bank operations in the implementation of monetary policy, and more generally of the attentiveness that the central bank has to exert to implement monetary policy and ensure the stability of the interbank market.

The timing of the model is summarized in Figure 2. At date 0 banks compete in prices on the loan market, choose reserve holdings, and raise deposits. After liquidity shocks materialize at date 1, the interbank market opens. At date 2 loans mature and remaining claims from deposits and the interbank market are settled.

2. THE STATUS QUO

In this section we characterize the equilibrium when all banks are identical. We start with noting two features of the model. First, bank runs never occur in this model. The illiquidity of loans together with $r^D > 0$ guarantees that depositors withdraw prematurely only if hit by liquidity shocks. Second, we assume that the loan market is sufficiently profitable (differentiated) for banks to borrow in the deposit and interbank markets. So, we can directly focus on the date 0 maximization problem.

With these considerations in mind, at date 0 each bank i chooses the loan rate r_i^L and the reserves R_i so as to maximize the following expected profit, where for simplicity the intertemporal discount factor is normalized to one:

$$\begin{aligned} \Pi_i = & (r_i^L - c)L_i + \int_0^{R_i} r^{LL}(R_i - x_i)f(x_i) dx_i - \int_{R_i}^{D_i} r^{IB}(x_i - R_i)f(x_i) dx_i \\ & - r^D D_i(1 - E(\delta_i)). \end{aligned} \tag{8}$$

The first term in equation (8) represents the profit from the loan market, the second term is the expected revenue from interbank lending at date 1 when the bank is in excess of reserves, the third term is the expected cost of refinancing at date 1 when the bank faces a shortage of reserves, and the fourth term is the expected repayment to depositors leaving their funds until date 2. Taken together, the last two terms represent bank i 's *financing costs*.

For expositional convenience and without loss of generality, we set $r^{LL} = 0$ and denote r^{IB} simply as r^I . (No qualitative result depends on this simplification, which also captures the stylized fact that the interbank market is relatively “passive” in that banks do not keep reserves to make profits, but only to protect themselves against liquidity shocks.) This simplifies equation (8) as follows:

$$\Pi_i = (r_i^L - c)L_i - \int_{R_i}^{D_i} r^I(x_i - R_i)f(x_i) dx_i - r^D D_i(1 - E(\delta_i)), \tag{9}$$

where the third term, $\int_{R_i}^{D_i} r^I(x_i - R_i)f(x_i) dx_i = \int_{R_i/D_i}^1 r^I(\delta_i D_i - R_i)f(\delta_i) d\delta_i$, indicates that if a bank's demand for liquidity, $x_i = \delta_i D_i$, exceeds its reserves R_i , the bank incurs the cost r^I on each unit of liquidity needed to satisfy depositors from the interbank market. Thus, as in Klein (1971), reserves are kept for precautionary reasons. In choosing the amount of reserves R_i at date 0, a bank trades off the cost of satisfying the expected liquidity needs at date 1, $\int_{R_i}^{D_i} r^I(x_i - R_i)f(x_i) dx_i$, with the cost r^D of raising more deposits and keeping more reserves at date 0. As a consequence, a bank's demand for reserves depends on the uncertainty about deposit withdrawals and on the costs incurred to borrow liquidity at date 1 and of keeping reserves initially. The more uncertain the date 1 demand for liquidity x_i and the more costly raising liquidity at date 1 (i.e., the higher r^I), the higher is the demand for reserves at date 0.

The following proposition characterizes the symmetric equilibrium in the status quo.

PROPOSITION 1: *The symmetric status quo equilibrium is characterized as follows:*

- (i) Each bank sets a loan rate $r_{sq}^L = l/[\gamma(\frac{N-1}{N})] + c_{sq}$, where $c_{sq} = c + \sqrt{r^I r^D}$;
- (ii) It has a loan market share $L_{sq} = l$;
- (iii) If $r^I > r^D$, it keeps reserves $R_{sq} = (\sqrt{r^I/r^D} - 1)L_{sq}$, and raises deposits $D_{sq} = L_{sq}\sqrt{r^I/r^D}$.

The differentiation on the loan market implies that the loan rate r_{sq}^L exceeds the total marginal cost c_{sq} via the mark up $l/[\gamma(\frac{N-1}{N})]$. This decreases with both the number of banks N and the loan substitutability parameter γ , while it increases with the level of loan demand l . The total marginal cost includes the loan lending cost c and the marginal financing cost $\sqrt{r^I r^D}$, that is, the sum of the expected cost of refinancing and of raising deposits.

Equilibrium reserve holdings R_{sq} balance the marginal benefit of reducing the expected cost of refinancing with the marginal cost of increasing deposits, as explained above, and they are positive as long as $r^I > r^D$. We restrict our attention to this plausible case. Reserves increase with the demand for loans L_{sq} and with the interbank refinancing cost r^I , while they decrease with the deposit rate r^D . The intuition is simple. When the demand for loans L_{sq} is high, banks face—*ceteris paribus*—higher deposit withdrawals at date 1 and wish to hold more reserves to satisfy them. Similarly, when the ratio r^I/r^D is high, banks prefer to keep more reserves initially, since this is less costly than obtaining the missing liquidity from the interbank market at date 1. In this sense, the ratio r^I/r^D can be defined as *the relative cost of refinancing*, which will help us later on to distinguish various scenarios for liquidity effects. It is a measure of how costly refinancing at date 1 is relative to raising deposits and reserves at date 0.⁶

Two further implications of Proposition 1 are important for comparing this equilibrium with the post-merger equilibrium in the next section. First, using the balance sheet equality (1), we can express equilibrium reserve holdings in terms of an *optimal reserve–deposit ratio* as

$$k_{sq} = \frac{R_{sq}}{D_{sq}} = \left(1 - \sqrt{\frac{r^D}{r^I}} \right). \tag{10}$$

Note that, whereas the equilibrium reserve holdings in Proposition 1 depend on the loan market outcome, the reserve–deposit ratio in equation (10) does not. To exploit this, in what follows we will mostly focus on this ratio. In practice, the ratios of liquid assets to customers’ sight deposits or of liquid assets to total assets are among the most frequently used indicators by banks to assess their own liquidity situation (see, e.g., European Central Bank, 2002, p. 22). Second, Proposition 1 implies the following corollary.

COROLLARY 1: *In the status quo equilibrium, each bank has liquidity risk $\phi_{sq} = \sqrt{r^D/r^I}$ and expected liquidity needs $\omega_{sq} = (r^D/2r^I)D_{sq} = L_{sq}/2\sqrt{r^D/r^I}$.*

The equilibrium liquidity risk ϕ_{sq} and the expected liquidity needs ω_{sq} are increasing in the deposit rate r^D and decreasing in the refinancing cost r^I . Banks keep low reserves when r^D is high and r^I is low, because raising more deposits is expensive while borrowing additional liquidity at date 1 is not. Thus, banks’ demand for reserves

6. If $r^L > 0$, the ratio would be $(r^{IB} - r^{LL})/(r^D - r^{LL})$.

decreases with the ratio r^D/r^L , and banks' liquidity risk and expected liquidity needs increase with it.

3. THE EFFECTS OF A MERGER ON INDIVIDUAL BANKS' BEHAVIOR

In this section we analyze what happens at the individual bank level when a merger takes place. The behavior of the merged banks changes in several ways. First, they can exchange reserves internally, which changes how they insure against liquidity risk. Second, this “internal money market” gives them a financing cost advantage, whose size is endogenously determined. Third, the merged banks may enjoy cost efficiencies that reduce their lending costs to β , where $\beta < 1$. Fourth, they gain market power in setting loan rates. All these factors affect banks' equilibrium balance sheets and, in turn, the demand and supply of liquidity. We begin with discussing how the merger modifies banks' reserve holdings, and then we turn to its effects on costs and loan market competition.

3.1 Internal Money Market and Choice of Reserves

We note first that the merger does not affect the optimal reserve–deposit ratio of the $N - 2$ competitors. As they have the same cost structure as in the status quo, they still choose their reserve–deposit ratios according to equation (10), that is, $k_c = k_{sq}$. This implies also that they have the same per-unit financing costs $\sqrt{r^L r^D}$ as in the status quo (from Proposition 1).

By contrast, the merged banks, say bank 1 and bank 2, choose a different reserve–deposit ratio, because the merger modifies the distribution of their liquidity shocks and also allows them to pool their reserves to meet the total demand for liquidity. Thus, as long as the two banks continue to raise deposits in two separate regions, the merger leaves room for an *internal money market* in which they can reshuffle reserves according to their respective needs. For simplicity, we assume a “perfect” internal money market, so that exchanging reserves internally involves no cost, but all qualitative results go through as long as the internal money market is less costly than the interbank one. Proceeding in this way is motivated by recent empirical research suggesting that internal capital markets function relatively efficiently (see, e.g., Houston, James, and Marcus 1997, Campello 2002, Graham, Lemmon, and Wolf 2002).

Let $x_m = \delta_1 D_1 + \delta_2 D_2$ be the total demand for liquidity of the merged banks at date 1, $R_m = R_1 + R_2$ be their total reserves, and $D_m = D_1 + D_2$ be their total deposits. The combined profits of the merged banks are then given by

$$\begin{aligned} \Pi_m = & (r_1^L - \beta c)L_1 + (r_2^L - \beta c)L_2 - \int_{R_m}^{D_m} r^L(x_m - R_m)f(x_m)dx_m \\ & - r^D[D_1(1 - E(\delta_1)) + D_2(1 - E(\delta_2))]. \end{aligned} \tag{11}$$

The first two terms in equation (11) represent the combined profits from the loan market, with β reflecting potential efficiency gains in the form of reduced loan lending costs, the third term is the total expected cost of refinancing, and the last one is the total expected repayment to depositors. The operation of the internal money market can be seen in the third term of equation (11), where the total demand for liquidity, $x_m = \delta_1 D_1 + \delta_2 D_2$, and reserves, $R_m = R_1 + R_2$, are pooled together.

A preliminary step before deriving the optimal reserve–deposit ratio of the merged banks is to understand their “deposit market policy.” Whether they raise equal or different amounts in both regions affects the distribution of the demand for liquidity x_m , and thus the size of the expected cost of refinancing. We have the following lemma.

LEMMA 1: *The merged banks raise an equal amount of deposits in each region, that is, $D_1 = D_2 = D_m/2$.*

Lemma 1 shows that the merged banks not only raise deposits in both regions, but they even do it symmetrically. Choosing equal amounts of deposits in both regions minimizes the variance of x_m and maximizes the benefits of diversification, thus reducing the expected refinancing cost. (We will come back to this point in Section 4 when studying the effect of the merger on aggregate liquidity demand.)

Given $D_1 = D_2$, the merged banks choose reserves R_m so as to maximize their combined profits in equation (11). Let $k_m = R_m/D_m$ be the reserve–deposit ratio for the merged banks and recall that k_{sq} is the one for banks in the status quo defined in equation (10). The following proposition compares these two ratios.

PROPOSITION 2: *The merged banks choose a lower reserve–deposit ratio than in the status quo ($k_m < k_{sq}$) if the relative cost of refinancing is higher than a threshold ρ ($r^I/r^D > \rho$), and a higher one otherwise.*

Proposition 2 contains the first main result of the paper indicating that the merged banks may have a higher or a lower optimal reserve–deposit ratio than individual banks. The result depends on the relative strength of two effects:

1. A diversification effect that reduces the probability of extreme shocks for the merged banks;
2. An internalization effect consisting of the possibility to use any unit of reserves to cover a deposit outflow at either of the banks that makes up the merged bank. This increases, *ceteris paribus*, the marginal value of one unit of reserves and has a positive impact on the demand for reserves.

Whether the merged banks choose a higher reserve–deposit ratio depends on which of the two effects dominates. Proposition 2 suggests that the internalization effect dominates when the relative cost of refinancing is low, whereas the diversification effect prevails when it is high. The intuition behind this result can be explained in

terms of the link between the marginal value of reserves, the initial level of reserves, and banks' ability to estimate future liquidity needs.⁷

The marginal value of reserves depends on the amount of reserves a bank has at date 0 and on the probability it will need more liquidity at date 1. The merged banks face less uncertainty about future liquidity needs than the individual banks, because their demand for liquidity is more concentrated around the mean. This means that at low initial level of reserves, a marginal increase of reserves is worth more to the merged banks than to the individual banks that are less certain of needing more liquidity at date 1. Conversely, when banks keep high levels of reserves, a marginal increase of reserves is worth less to the merged banks than to the individual banks, since the merged banks know they will be less likely to need liquidity at date 1. Whether the merged banks increase their reserve holdings compared to before merging depends on the relative cost of refinancing r^I/r^D . When this ratio is low, all banks keep low reserves because refinancing is not very expensive. Then the merged banks increase their reserves relative to before merging, since they have a higher marginal value of further reserve units. The opposite happens when r^I/r^D is high. In this case reserves are high, and the merged banks value further increases of reserves less than individual banks, because they are more confident that they will not need them.

The readjustment of the merged banks' reserve holdings changes also their refinancing costs relative to the status quo, and we have the following.

COROLLARY 2: The merged banks have lower financing costs than the competitors.

This cost advantage for the merged banks is endogenous to the model in that it is determined not only by diversification, but also by the optimal reserve readjustment. In this sense, this result provides a new bank-specific motive to merge, in addition to the well-known market power and diversification motives.

To sum up, the possibility for merged banks to estimate more precisely their liquidity needs allows them to better assess the reserve–deposit ratio they should hold and adjust it accordingly. Furthermore, the possibility for the merged banks to exchange reserves in the internal money market implies that banks can benefit from scope economies in their liquidity management by raising deposits in two imperfectly correlated deposit markets. This result is consistent with Hughes, Land, Mester and Moon (1996), who find that banks active in imperfectly correlated deposit markets—especially as a result of consolidation—can reduce the cost of controlling liquidity risk by appropriately adjusting deposit collection and reserve holdings. In this respect, our result is also related to Kashyap, Rajan, and Stein (2002), who show that combining the activities of lending and deposit taking produces synergies that allow banks to reduce the volume of liquid assets that banks need to hold to satisfy their customers' unexpected demands. However, whereas in their paper such an advantage in providing liquidity arises as a consequence of two imperfectly correlated markets

7. We thank Loretta Mester for suggesting this explanation.

on different sides of the balance sheet, in our model it emerges from two imperfectly correlated markets on the same side of the balance sheet.

3.2 Choice of Loan Rates and Balance Sheets

We now examine how the merger modifies the loan market equilibrium and banks' balance sheets. The effect of the merger on loan rates depends on how it affects banks' market power and cost structures. As already noticed, competitors have the same total costs as in the status quo. By contrast, the total costs of the merged banks change. As stated in Corollary 2, their financing costs are lower than competitors'. Furthermore, their lending costs reach βc , where the parameter $\beta \leq 1$ represents the potential non-financial efficiency gains that the merger induces for granting loans. The lower the parameter β , the greater are the efficiency gains. The idea is to include, for example, the possibility for economies of scale, which are often put forward by bank managers in favor of mergers and have been questioned in the literature, as we discuss in Section 5.⁸

The following proposition describes the post-merger equilibrium with symmetric behavior within the "coalition" (merger) and among competitors.

PROPOSITION 3: *The post-merger equilibrium with $r_1^L = r_2^L = r_m^L$ and $r_i^L = r_c^L$ for $i = 3, \dots, N$ is characterized as follows:*

- (i) *Each merged bank sets a loan rate $r_m^L = [(2N - 1)/(N - 2)](l/2\gamma) + [(N - 1)/2N]c_c + [(N + 1)/2N]c_m$, and each competitor sets $r_c^L = [(N - 1)/(N - 2)]l/\gamma + [(N - 1)/N]c_c + (1/N)c_m$;*
- (ii) *The merged banks have a total loan market share $L_m = [(2N - 1)/N]l + \gamma[(N - 1)(N - 2)/N^2](c_c - c_m)$, and each competitor has $L_c = [(N - 1)^2/N(N - 2)]l - \gamma[(N - 1)/N^2](c_c - c_m)$;*
- (iii) *The merged banks raise total deposits $D_m = [1/(1 - k_m)]L_m$, and each competitor raises $D_c = [1/(1 - k_c)]L_c$; where c_m, c_c are the total marginal costs of the merged banks and of the competitors, and k_m and k_c are their respective optimal reserve–deposit ratios.⁹*

Since banks compete in strategic complements, the merged banks drive the loan rate movements in the market and the competitors move in the same direction. The effect of a merger on loan rates depends on the relative strength of a *market-power effect* and a *cost-efficiency effect*. The first refers to the higher market power banks enjoy after a merger because of the lower number of active banks (which reduces from N to $N - 1$). The second derives from the lower total marginal costs c_m that the merged banks enjoy relative to competitors. Post-merger equilibrium loan rates

8. We could also allow for $\beta > 1$, in which case the merger would even lead to diseconomies. Already the market power of merged banks tends to increase loan rates, and $\beta > 1$ would only strengthen this effect. So, none of our results would be qualitatively altered by further generalizing β .

9. The expressions for c_m, c_c are in the proof of this proposition; those for k_m and k_c are, respectively, in the proof of Proposition 2 and in equation (10).

increase when the merger induces small cost advantages relative to the increase in market power, whereas they decrease otherwise.

Loan market shares across banks change in line with loan rates. As the merged banks change their loan rates by more than competitors, their total loan market share shrinks when loan rates increase and it expands otherwise, that is, $L_m < 2L_{sq} < 2L_c$ when $r_m^L > r_c^L$, and $L_m > 2L_{sq} > 2L_c$ otherwise.

The modification of loan market shares together with the change in the optimal reserve–deposit ratio described in Proposition 2 determines the effects on the size of banks’ balance sheets (as measured by the amount of deposits). In the present setup the merger breaks the symmetry in banks’ balance sheets. Whereas in the status quo all banks have the same deposits D_{sq} , the merged banks have now in general different deposit sizes than competitors, that is, $D_m/D_c \neq 2$. This is what we assume here, although the opposite could also happen: starting from a situation of an asymmetric banking system, the merger could reduce the asymmetry among banks and make the system more homogenous.

3.3 Individual Banks’ Liquidity Risk and Expected Needs

The effects of the merger on both banks’ reserve holdings and loan competition affect also banks’ liquidity risks and expected liquidity needs. The results for competitor banks are quite straightforward. As they follow the same optimal reserve rule as in the status quo, they face the same liquidity risk $\phi_c = \phi_{sq} = \sqrt{r^D/r^I}$ (see Corollary 1). Their expected liquidity needs, however, change with their balance sheet, as $\omega_c = (r^D/2r^I)D_c$. The merged banks experience more far reaching changes in probability of facing a liquidity shortage and in the size of the expected needs.

COROLLARY 3: *The merged banks have lower liquidity risk than a single bank in the status quo.*

This result derives directly from the readjustment of the merged banks’ reserve holdings. As stated in Proposition 2, when the relative cost of refinancing is below the threshold ρ , the merged banks increase their reserve–deposit ratio and their liquidity risk goes down. In the other case, although they choose a lower reserve–deposit ratio than in the status quo, they still keep it sufficiently high to decrease the liquidity risk. This effect is so strong that the liquidity risk of the merged banks is not only lower than the risks of two banks in the status quo, but it is even lower than that of a single bank.

COROLLARY 4: *The merged banks have lower expected liquidity needs than in the status quo if $D_m/D_{sq} < h$, where $2 < h \leq 4$, and higher ones otherwise.*

The merger changes the merged banks’ expected needs for three reasons. First, it creates the internal money market, which reduces, *ceteris paribus*, expected liquidity needs. Second, the merger modifies the merged banks’ optimal reserve–deposit ratio,

which reduces *ceteris paribus* expected liquidity needs when the relative cost of refinancing is low. Third, the merger changes the merged banks' deposits, and hence the size of their demand for liquidity. Corollary 4 shows that the first effect dominates unless cost advantages (efficiency gains and reduced financing costs) and competition in the loan market (degree of loan differentiation γ and number of banks N) are so strong that the merged banks increase their balance sheets substantially relative to two banks in the status quo. From an empirical perspective, such a strong balance-sheet expansion seems to be a less plausible scenario.

4. THE EFFECTS OF A MERGER ON AGGREGATE LIQUIDITY

Now that we have seen how a merger affects the behavior of individual banks, we can turn to its implications for the banking system as a whole. To see this, we analyze how changes in banks' reserve holdings and in loan market competition modify the aggregate supply and demand of liquidity, as represented, respectively, by the sum of all banks' reserves and of their demands for liquidity at date 1 when shocks materialize.

We identify two channels. The first one we call *reserve channel*, as it works through changes in reserve holdings. When looking at the system as a whole, the distinction between the internal money market of the merged banks and the interbank market is blurred, and the total supply of liquidity is composed of the sum of all banks' reserve holdings. Nevertheless, the existence of the internal money market affects the total supply of liquidity through the change in the reserve holdings of the merged banks. The second channel is an *asymmetry channel*, which affects the distribution of the aggregate liquidity demand. This channel originates in the asymmetry of balance sheets across banks, which—as shown above—depends on both the different amounts of reserves and the different loan market shares that banks have after the merger.

We start with analyzing each of the two channels in isolation. Then we examine how they interact in determining aggregate liquidity risk and expected aggregate liquidity needs.

4.1 *Asymmetry Channel without Internal Money Market*

To isolate the working of the asymmetry channel, we assume for a moment that the merged banks cannot make use of the internal money market. In this case, they do not have any financing cost advantages, and they choose the same optimal reserve rule as their competitors. As a consequence, the asymmetry in banks' balance sheets originates only from the different distribution of market shares resulting from loan competition.

As all banks continue to choose reserves according to equation (10) and as the aggregate demand for loans is inelastic, the merger does not affect the total amounts of reserves and deposits, thus leaving the aggregate supply of private liquidity

unchanged. The asymmetry of banks' balance sheets, however, modifies the aggregate liquidity demand, which changes from $X_{sq} = \sum_{i=1}^N \delta_i D_{sq}$ in the status quo to $X_m = \delta_1(D_m/2) + \delta_2(D_m/2) + \sum_{i=3}^N \delta_i D_c$ after the merger. Both X_{sq} and X_m are weighted sums of N uniform random variables, but in the first case weights are equal and in the second case they differ (according to deposit sizes $D_m/2$ and D_c). This brings us to the main result about the asymmetry channel.

PROPOSITION 4: *Suppose the merged banks do not exchange reserves internally. Then, the aggregate liquidity effects of the merger are as follows:*

- (i) *The merger decreases aggregate liquidity risk if the relative cost of refinancing is below a threshold σ ($r^I/r^D < \sigma < \rho$), and increases it otherwise;*
- (ii) *The merger always increases expected aggregate liquidity needs.*

The intuition behind Proposition 4 is similar to that one behind Lemma 1. Moving from a uniformly weighted sum of random variables (in the status quo) to a heterogeneously weighted sum of random variables (after merger) increases the variance of the total sum. Thus, as Figure 3 illustrates, the distribution of X_{sq} gives lower probability to extreme events—very low and very high realizations of the aggregate liquidity demand—than that of X_m .

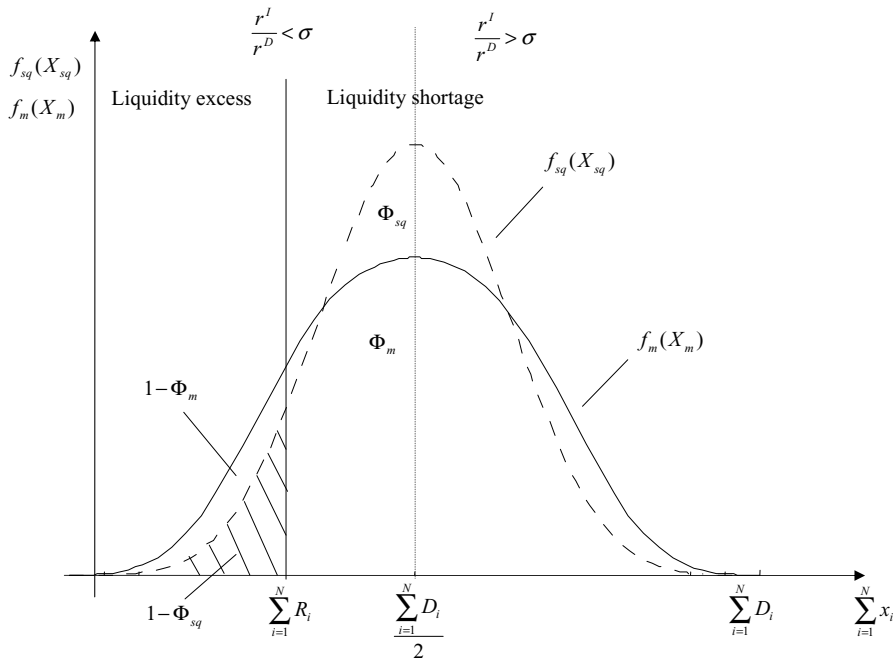


FIG. 3. Aggregate Liquidity Risk Before Merger, Φ_{sq} , and After Merger, Φ_m .

This change in the distribution of X_m reduces the aggregate liquidity risk if the relative cost of refinancing is low (below the threshold σ), because it increases the probability that the aggregate liquidity demand is below the total private supply. This is illustrated in Figure 3, where total reserves—indicated by the vertical line $\sum_{i=1}^N R_i$ —are low and the area $1 - \Phi_m$ is larger than the diagonally striped area $1 - \Phi_{sq}$. The opposite happens when the relative cost of refinancing is high.

Proposition 4 also states that the merger always increases the expected amount of public liquidity needed. The reason is that the expected aggregate liquidity needs depend not only on the frequency with which aggregate liquidity demand exceeds aggregate supply, but also on the magnitude of each excess. As noted earlier, the merger increases the variance of the distribution of X_m and thus the probability of events with very low and very high demands. If banks do not hold reserves, these increases offset each other and the expected aggregate liquidity needs are the same before and after the merger. By contrast, when banks hold positive reserves, they can cover the events with low aggregate liquidity demand. Hence, the higher probability of extreme events with high aggregate liquidity demand is not outweighed any more by the higher frequency of low demand events, and the expected aggregate liquidity needs grow.

Note that the results of Proposition 4 crucially depend on the fact that we are focusing on mergers that lead to more asymmetry in banks' balance sheets, like those among large banks. Of course, mergers may also have the opposite effect of making banks more symmetric. This could happen, for example, when mergers involve smaller banks. In such a case, the functioning of the asymmetry channel is reversed and mergers reduce expected aggregate liquidity needs. We need to keep this in mind when discussing our results further below.

4.2 Interaction with the Reserve Channel

In this section we reintroduce the possibility for the merged banks to use the internal money market. We first analyze how this affects aggregate liquidity through the reserve channel. Denote as

$$K_m = \frac{R_m + \sum_{i=3}^N R_c}{D_m + (N - 2)D_c} = \frac{k_m D_m + \sum_{i=3}^N k_c D_c}{D_m + (N - 2)D_c} \tag{12}$$

the aggregate reserve–deposit ratio after the merger. Since competitors choose the same ratio as in the status quo ($k_c = k_{sq}$), the change in K_m is solely determined by the change in the merged banks' reserve–deposit ratio. Hence, it follows from Proposition 2 that K_m increases when the relative cost of refinancing is relatively low (because then $k_m > k_{sq}$), whereas it decreases otherwise. The following lemma describes how the change in the aggregate reserve–deposit ratio alone affects aggregate liquidity.

LEMMA 2: *Suppose the merger does not cause any asymmetry in banks' balance sheets ($D_m = 2D_c$). Then, it decreases aggregate liquidity risk and expected aggregate liquidity needs if the relative cost of refinancing is below ρ ($r^I/r^D < \rho$), and it increases them otherwise.*

When the merger does not generate asymmetry across banks' balance sheets, it affects aggregate liquidity only through the reserve channel. The aggregate supply of private liquidity changes, whereas the aggregate liquidity demand remains the same. Thus, the merger reduces both aggregate liquidity risk and expected aggregate liquidity needs when the aggregate supply of private liquidity increases through the higher reserve–deposit ratio of the merged banks. The opposite happens when the aggregate liquidity supply falls.

When the merger generates the internal money market and modifies bank sizes, both the asymmetry and the reserve channel are at work. Depending on the size of the relative cost of refinancing, the two channels can reinforce or offset each other. Therefore, we consider the cases of high and low relative cost of refinancing separately.

PROPOSITION 5: *If the relative cost of refinancing is above ρ ($r^I/r^D > \rho$), the merger increases both aggregate liquidity risk and expected aggregate liquidity needs.*

When the relative cost of refinancing is rather high, the asymmetry channel and the reserve channel work in the same direction. The asymmetry channel increases the variance of the aggregate liquidity demand, and the reserve channel reduces the aggregate liquidity supply through the lower reserve holdings of the merged banks. Both these effects make the system more vulnerable to liquidity shortages and thus more dependent on public liquidity provision.

PROPOSITION 6: *If the relative cost of refinancing is below ρ ($r^I/r^D < \rho$), then the following holds:*

- (i) *There exists a critical level of the relative cost of refinancing $g \in (\sigma, \rho)$ such that the merger reduces aggregate liquidity risk if the cost of refinancing is below such critical level, and increases it otherwise;*
- (ii) *For any small level of asymmetry induced by the merger, there exists a set G of values of the relative cost of refinancing, with $G \subset (1, \rho)$, for which the merger reduces expected aggregate liquidity needs.*

When the cost of refinancing is relatively low, the reserve and the asymmetry channels drive aggregate liquidity in opposite directions, and the net effect depends on their relative strength. As shown in Lemma 2, the reserve channel reduces both aggregate liquidity risk and expected liquidity needs. This occurs because the banking system has more reserves in aggregate through the higher reserve holdings of the merged banks. As stated in Proposition 4, however, the asymmetry channel always increases expected aggregate liquidity needs, whereas it reduces aggregate liquidity risk only if the relative cost of refinancing is sufficiently low.

Thus, when the two channels interact, the merger reduces aggregate liquidity risk for a larger range of parameter values than in Proposition 4, where only the asymmetry channel is active. Similarly, it increases aggregate liquidity risk in a larger range of parameter values than in Lemma 2, where only the reserve channel is present.

As for the expected aggregate liquidity needs, the reserve channel dominates when the asymmetry induced by the merger is sufficiently small. Thus, there is a range of values of the relative cost of refinancing for which the merger reduces expected aggregate liquidity needs. The larger the asymmetry in banks' balance sheets, the larger is this range of parameters in which the merger increases expected aggregate liquidity needs. Taken together, the results in Proposition 6 suggest that central banks have to be more attentive to the liquidity fluctuations of the interbank market and intervene more often after a consolidation process that leads to higher aggregate liquidity risk and higher expected liquidity needs.

How relevant are these different scenarios for changes in aggregate liquidity? One way to proceed is to associate the level of the relative cost of refinancing with different countries or financial systems. For example, in industrial countries with relatively sizable and developed financial systems one would expect this cost to be rather low. In contrast, in developing or emerging countries with less developed financial systems this cost may be quite high. Then, Proposition 5 suggests that in the latter group of countries bank consolidation may lead to a deterioration of aggregate liquidity. Proposition 6 indicates instead that the impact of mergers on aggregate liquidity in industrial countries crucially depends on whether the reserve or the asymmetry channel dominates. The asymmetry channel may dominate when consolidation takes the form of mergers between large banks leading to a "polarization" of the banking system. Table 1 suggests that something like this seems to have happened in a number of industrial countries during the 1990s. For example, in Belgium, Canada, France, the Netherlands, and Sweden, consolidation enlarged substantially the share of the largest players, thus increasing the asymmetry among banks. Differently, in countries like Australia and Germany, the weight of the largest banks hardly changed. So it may be possible that the reserve channel may have dominated in those countries, thus leading to an improvement in aggregate liquidity. A similar result may have occurred in Japan and the United Kingdom, where—according to Table 1—consolidation has even led to a more symmetric banking system and thus to a reversed functioning of the asymmetry channel.

Note that we primarily address the structural effects of bank mergers on loan competition, reserve holdings, and aggregate liquidity, but in practice business cycles may affect some of our variables. In particular, the relative cost of refinancing is affected by trading conditions in the interbank market and the level of interest rates, and it may behave pro-cyclically. This implies that bank mergers involving large banks may affect reserve holdings and private liquidity more negatively in upturns than in downturns. Our results above have therefore to be interpreted as the "average" (or structural) effects of mergers over time. It would be interesting to extend our model in future research to explicitly cover macroeconomic features and analyze in depth how reserve holdings and aggregate liquidity change over the business cycle.

5. THE RELATIONSHIP BETWEEN COMPETITION AND AGGREGATE LIQUIDITY

We now discuss more in detail how loan market competition and reserve choices interact in determining loan rates and aggregate liquidity (for simplicity, here interpreted only as expected aggregate liquidity needs), and how liquidity effects relate to competition effects.

At the individual bank level, the loan market equilibrium affects banks' reserve holdings (in absolute terms) by determining the amount of deposits required to finance loans, and hence the size of liquidity demands at any given level of reserves. Equilibrium reserve holdings determine banks' financing costs—the sum of the expected cost of refinancing and of the expected repayment to depositors—and thereby influence the loan market equilibrium. At the aggregate level, loan market competition affects the degree of asymmetry in banks' balance sheets through the distribution of equilibrium loan market shares.

Table 2 summarizes the possible effects of mergers on both loan rates r^L and expected aggregate liquidity needs Ω , as described in Propositions 3, 5, and 6. The rows of the table indicate whether a merger is characterized by low or high efficiency gains in terms of both reduced loan provision costs and lower financing costs (c_m/c_c high or low); the two columns show the cases of high and low relative cost of refinancing r^I/r^D .

As Table 2 shows, the model predicts several scenarios, depending on the value of the parameters. The effect of mergers on expected aggregate liquidity needs is ambiguous when the relative cost of refinancing is low, whereas it is always negative when the relative cost of refinancing is high. Concerning competition, mergers increase loan rates when the cost efficiency effect, as measured by the ratio c_m/c_c , is small relative to the increased market power and vice versa, decrease loan rates when cost efficiencies dominate.

What can we say about the plausibility of the different scenarios displayed in Table 2? As already indicated above, one may associate for example low refinancing

TABLE 2
EFFECTS OF A MERGER ON LOAN RATES AND EXPECTED AGGREGATE LIQUIDITY NEEDS

	$\frac{r^I}{r^D}$ low	$\frac{r^I}{r^D}$ high
	I	III
$\frac{c_m}{c_c}$ high	$\Omega \uparrow \downarrow$ $r^L \uparrow$	$\Omega \uparrow$ $r^L \uparrow$
	II	IV
$\frac{c_m}{c_c}$ low	$\Omega \uparrow \downarrow$ $r^L \downarrow$	$\Omega \uparrow$ $r^L \downarrow$

costs with industrial countries and high refinancing costs with developing or emerging countries. Moreover, one may relate the magnitude of efficiency gains to the size of mergers. Even if there is an ongoing debate in the literature on whether efficiency gains (and, in particular, scale economies) exhaust at large or small sizes of output, the empirical consensus seems still to be that mergers between small banks produce larger efficiency gains than mergers between large banks.¹⁰

One plausible scenario in industrial countries (low r^I/r^D) is therefore the occurrence of mergers between large banks leading to higher loan rates and expected aggregate liquidity needs, as they do not realize sufficient efficiency gains and induce greater asymmetry in the banking system (one case in cell I). Differently, the occurrence of mergers between small banks in industrial countries is likely to reduce both loan rates and expected aggregate liquidity needs, as smaller mergers may realize more efficiency gains relative to the increase in market power and make the banking system more homogenous (one case in cell II). For developing countries (high r^I/r^D), cells III and IV suggest that mergers would always increase expected aggregate needs, whereas the effect on loan rates may still depend on their sizes.

The interesting features of these results are that mergers are likely, *ceteris paribus*, to increase expected aggregate liquidity needs more in developing countries than in industrial ones, as they lead to lower reserve holdings for higher cost of refinancing; and that there is more complementarity between competition and liquidity in industrial countries than in developing ones. In terms of policy implications, these results suggest that policies aiming at promoting loan market competition may also prevent the adverse effects of consolidation for interbank liquidity in industrial countries, but not necessarily in developing countries.

6. CONCLUSIONS

This paper analyzes the impact of bank mergers on credit market competition, reserves, and banking system liquidity. A merger creates an internal money market, which modifies merged banks' optimal choice of reserves holdings, either decreasing them through a diversification effect or—more surprisingly—increasing them through an internalization effect. In both situations, merged banks benefit from a better estimate of future shocks and scope economies in their liquidity management, and they lower their financing costs and liquidity risk.

The change in merged banks' reserve holdings, together with the change in the size of banks' balance sheets due to altered loan competition, affect the functioning of the interbank market. Changes in reserve holdings modify the aggregate supply of

10. A substantial amount of empirical research has been spent on measuring the efficiency gains generated by bank mergers, but results are not unanimous (see, e.g., the surveys of Rhoades 1994, 1998, Carletti, Hartmann, and Spagnolo 2002). Whereas the mainstream literature suggests that banks exhaust potential scale economies at modest levels of size (see, e.g., Berger, Hanweck, and Humphrey 1987, Berger and Humphrey 1991, Wheelock and Wilson 2001), other studies (e.g., Berger and Mester 1997, Hughes, Mester and Moon 2001) find that there are scale economies also at large balance sheet sizes if one takes changes in risk into account.

private liquidity, while increased balance-sheet asymmetry raises aggregate liquidity needs by altering the distribution of the aggregate liquidity demand. These reserve and asymmetry channels can work in the same or in opposite directions, depending on the cost of refinancing in the money market as compared to the cost of financing through retail deposits. We conclude that mergers are more likely to increase aggregate liquidity needs when they involve large banks leading to a “polarization” of the banking system. Moreover, the risk of adverse liquidity effects of bank consolidation is likely to be more relevant when the ratio of interbank to deposit funding costs is high. In this case there is also a lower complementarity between competition and liquidity, so that the effects of consolidation on loan rates and aggregate liquidity do not necessarily go hand in hand. These results have important implications for central banks’ money market operations.

The model implies some empirical hypotheses, which would be interesting to test in future research. While the competition effects of bank mergers are already quite well covered in the empirical literature, the same does not apply to the liquidity effects. At the individual level it would be interesting to estimate the effects of mergers on reserve holdings, and in particular the role of refinancing costs for the sign of reserve changes. At the aggregate level, it would be important to examine how asymmetry in bank sizes relates to liquidity fluctuations. Moreover, it could be tested whether countries with relatively high refinancing costs experiencing banking consolidation display a deterioration in aggregate liquidity, whereas others do not. Finally, it could be interesting to examine econometrically whether countries with greater bank competition face larger or smaller aggregate liquidity fluctuations.

Some features of the model deserve further discussion. The interbank market works in a very simple way. In the ultra-short interbank market, the central bank adjusts the liquidity supply to accommodate changes in the aggregate demand, and banks can always meet the repayment to depositors without suffering any liquidity crisis. In a similar spirit, long-term loans are totally illiquid, or, equivalently, the costs of liquidation are higher than the relative cost of refinancing. This framework allows us to focus on pure liquidity issues, and isolate reserve management from other considerations. An interesting extension of this model would be to analyze the functioning of other, longer-term interbank markets, where the central bank would not be active and banks could modify the liquidity supply only by selling their long-term assets. We leave this for future research.

APPENDIX

PROOF OF PROPOSITION 1: Using Leibniz’s rule and equation (1), from (9) we obtain the first order conditions with respect to the choice variables r_i^L and R_i :

$$\frac{\partial \Pi_i}{\partial r_i^L} = L_i + (r_i^L - c) \frac{\partial L_i}{\partial r_i^L} - \left[\frac{r^I}{2} \frac{L_i^2 + 2L_i R_i}{(L_i + R_i)^2} + \frac{r^D}{2} \right] \frac{\partial L_i}{\partial r_i^L} = 0, \quad \text{for } i = 1 \dots N, \quad (A1)$$

$$\frac{\partial \Pi_i}{\partial R_i} = r^D(L_i + R_i)^2 - r^I L_i^2 = 0, \quad \text{for } i = 1 \dots N. \tag{A2}$$

Solving equation (14) for R_i gives

$$R_i = \left(\sqrt{\frac{r^I}{r^D}} - 1 \right) L_i. \tag{A3}$$

Solving equation (A1) for r_i^L in a symmetric equilibrium where $r_i^L = r_{sq}^L$ for $i = 1 \dots N$ after substituting equations (2) and (A3) gives

$$l + \left(r_{sq}^L - c - \sqrt{r^I r^D} \right) \left(-\gamma \frac{N-1}{N} \right) = 0,$$

from which r_{sq}^L and c_{sq} follow. Substituting then r_{sq}^L in equation (2) gives L_{sq} , and through equation (A3) R_{sq} . Substituting R_{sq} and L_{sq} in equation (1), we obtain D_{sq} . \square

PROOF OF COROLLARY 1: Solving equations (3) and (4) gives $\phi_i = 1 - (R_i/D_i)$ and $\omega_i = (R_i)^2/2D_i - R_i + D_i/2$. Substituting the expressions for R_{sq} and D_{sq} , we obtain ϕ_{sq} and ω_{sq} as in the corollary. \square

PROOF OF LEMMA 1: We proceed in two steps. First, we show that the variance of the liquidity demand x_m of the merged banks is minimized when deposits are raised symmetrically in the two regions. Second, we show that the expected liquidity needs of the merged banks (and therefore their refinancing costs) are lower when deposits are symmetric.

Step 1. Define the liquidity demand of the merged banks as

$$x_m = \delta_1 \alpha D_m + \delta_2 (1 - \alpha) D_m,$$

where $\alpha \in [0, 1]$ indicates the fraction of deposits that the merged banks raise in one region and $(1 - \alpha)$ the fraction they raise in the other region. Since δ_1 and δ_2 are independent and $\text{Var}(\delta_1) = \text{Var}(\delta_2)$, the variance of x_m is simply

$$\begin{aligned} \text{Var}(x_m) &= \alpha^2 D_m^2 \text{Var}(\delta_1) + (1 - \alpha)^2 D_m^2 \text{Var}(\delta_2) \\ &= \text{Var}(\delta_1) [\alpha^2 D_m^2 + (1 - \alpha)^2 D_m^2]. \end{aligned}$$

Differentiating it with respect to α , we obtain

$$\frac{\partial \text{Var}(x_m)}{\partial \alpha} = 2D^2 \text{Var}(\delta_1) (2\alpha - 1) = 0,$$

which has a minimum at $\alpha = 1/2$.

Step 2. Define now the liquidity demand of the merged banks as

$$x_{ma} = \delta_1 \alpha D_m + \delta_2 (1 - \alpha) D_m,$$

when $\alpha \neq 1/2$, and as

$$x_{ms} = \delta_1 \frac{D_m}{2} + \delta_2 \frac{D_m}{2}$$

when $\alpha = 1/2$. Applying the general formula in Bradley and Gupta (2002) to our case, the density functions of x_{ma} and x_{ms} can be written as (assume $\alpha < 1/2$ without loss of generality):

$$f_{ma}(x_{ma}) = \begin{cases} \frac{x_{ma}}{\alpha(1-\alpha)D_m^2} & \text{for } x_{ma} \leq \alpha D_m \\ \frac{1}{(1-\alpha)D_m} & \text{for } \alpha D_m < x_{ma} \leq (1-\alpha)D_m \\ \frac{D_m - x_{ma}}{\alpha(1-\alpha)D_m^2} & \text{for } x_{ma} > (1-\alpha)D_m, \end{cases}$$

$$f_{ms}(x_{ms}) = \begin{cases} \frac{4x_m}{D_m^2} & \text{for } x_{ms} \leq D_m/2 \\ \frac{4(D_m - x_{ms})}{D_m^2} & \text{for } x_{ms} > D_m/2. \end{cases} \tag{A4}$$

Since $\alpha < 1/2$, $f_{ma}(x_{ma})$ is steeper than $f_{ms}(x_{ms})$ both for $x_{ma} \leq \alpha D_m$ and for $x_{ma} > (1 - \alpha)D_m$. This implies that the two density functions do not cross in these intervals, whereas they do it in two points in the interval $\alpha D_m < x_{ma} \leq (1 - \alpha)D_m$. Given that they are symmetric around the same mean $D_m/2$ with $\text{Var}(x_{ma}) > \text{Var}(x_{ms})$, it is:

$$\begin{aligned} F_{ma} &> F_{ms} && \text{for } R_m < D_m/2, \\ F_{ma} &< F_{ms} && \text{for } R_m > D_m/2, \end{aligned} \tag{A5}$$

where $F_{ma} = \Pr(x_{ma} < R_m)$ and $F_{ms} = \Pr(x_{ms} < R_m)$.

Denote now as ω_{ma} and ω_{ms} the expected liquidity needs of the merged banks with asymmetric deposits, and symmetric deposits, respectively. We have

$$\begin{aligned} \omega_{ma} - \omega_{ms} &= \int_{R_m}^{D_m} (x_{ma} - R_m) f_{ma}(x_{ma}) d(x_{ma}) - \int_{R_m}^{D_m} (x_{ms} - R_m) f_{ms}(x_{ms}) d(x_{ms}) \\ &= \int_{R_m}^{D_m} x_{ma} f_{ma}(x_{ma}) d(x_{ma}) - \int_{R_m}^{D_m} x_{ms} f_{ms}(x_{ms}) d(x_{ms}) \\ &\quad - R_m(1 - F_{ma}(R_m)) + R_m(1 - F_{ms}(R_m)). \end{aligned} \tag{A6}$$

Differentiating equation (A6) with respect to R_m gives

$$\begin{aligned} d(\omega_{ma} - \omega_{ms})/dR_m &= -R_m f_{ma}(R_m) + R_m f_{ms}(R_m) - (1 - F_{ma}(R_m)) \\ &\quad + R_m f_{ma}(R_m) + (1 - F_{ms}(R_m)) - R_m f_{ms}(R_m) \\ &= F_{ma}(R_m) - F_{ms}(R_m). \end{aligned}$$

From equation (A5) it follows $d(\omega_{ma} - \omega_{ms})/dR_m > 0$ for $R_m < D_m/2$ and $d(\omega_{ma} - \omega_{ms})/dR_m < 0$ otherwise. This, along with $\omega_{ma} - \omega_{ms} = 0$ both for $R_m = 0$ and for $R_m = D_m$ implies $\omega_{ma} - \omega_{ms} > 0$ for all $R_m \in [0, D_m]$. \square

PROOF OF PROPOSITION 2: The demand for liquidity of the merged banks, $x_m = \delta_1(D_m/2) + \delta_2(D_m/2)$, has density function as in equation (A4). Using Leibniz’s rule, the equality $D_m = R_m + L_1 + L_2$, and the ratio $k_m = R_m/D_m$, from equation (11) we can express the first-order condition $\partial \Pi_m / \partial R_m = 0$ as

$$\begin{cases} \frac{8}{3}k_m^3 - 4k_m^2 + 1 = \frac{r^D}{r^I} & \text{for } k_m \leq 1/2 \\ \frac{8}{3}(1 - k_m)^3 = \frac{r^D}{r^I} & \text{for } k_m > 1/2. \end{cases} \tag{A7}$$

The term on the left-hand side (LHS) of the equalities is the marginal benefit of increasing the reserve–deposit ratio, that is the reduction in the expected need of refinancing induced by a marginal increase of the reserve ratio. The term on the right-hand side (RHS) of the equalities is the ratio between the marginal cost of raising reserves r^D and the marginal cost of refinancing r^I . From equation (A7), we obtain:

$$k_m = \begin{cases} z(r^I, r^D) & \text{for } r^I \leq 3r^D \\ 1 - \sqrt[3]{\frac{3}{8} \frac{r^D}{r^I}} & \text{for } r^I > 3r^D, \end{cases} \tag{A8}$$

where $z(r^I, r^D)$ is the solution of the equation $z^3 - (3/2)z^2 + (3/8)[1 - (r^D/r^I)] = 0$ in the interval $(0, 1/2]$ increasing in the ratio r^I/r^D . Since $f(0) > 0$, $f(1/2) < 0$ and $f'(z) < 0$, $z(r^I, r^D)$ is the unique real solution.

To compare k_m with k_{sq} , we rearrange k_{sq} given in equation (10) as

$$(1 - k_{sq})^2 = \frac{r^D}{r^I}, \tag{A9}$$

where, as before, the LHS is the marginal benefit of increasing the reserve–deposit ratio and the RHS is the ratio between the marginal cost of raising deposits and holding reserves r^D and the marginal cost of refinancing r^I .

Denote as $f(k_m)$ the LHS of equation (A7) and as $f(k_{sq})$ the LHS of equation (A9). Plotting $f(k_m)$ and $f(k_{sq})$ for k_{sq} and k_m between 0 and 1, we get Figure A1.

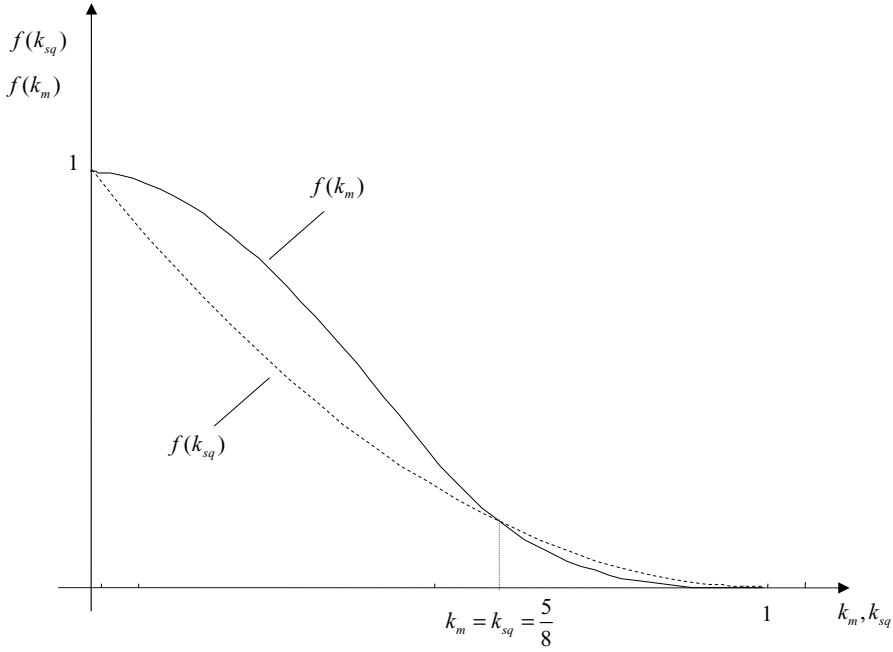


FIG. A1. Marginal Benefits of Higher Reserve-Deposit Ratios for the Merged Banks, $f(k_m)$, and Banks in the Status Quo, $f(k_{sq})$.

The curves $f(k_m)$ and $f(k_{sq})$ cross only once at $k_{sq} = k_m = 5/8$. Substituting this value in equation (A7) or (A9) gives $k_{sq} = k_m$ when $r^I/r^D = 64/9 \equiv \rho$. Thus, $k_m > k_{sq}$ if $r^I/r^D < \rho$, and $k_m < k_{sq}$ otherwise. \square

PROOF OF COROLLARY 2: From the last two terms in equation (8), we can express the financing costs of competitors as

$$\frac{r^I}{2} \frac{L_c^2}{(R_c + L_c)} + \frac{r^D}{2} (R_c + L_c). \tag{A10}$$

Using $R_c/D_c = k_c$ and $L_c/D_c = 1 - k_c$ in equation (A10) and rearranging terms, we obtain

$$\frac{r^I(1 - k_c)^2 + r^D}{2(1 - k_c)}. \tag{A11}$$

Analogously, from the last two terms in equation (11), using $R_m/D_m = k_m$ and $L_m/D_m = 1 - k_m$, we obtain the financing costs of the merged banks as

$$\begin{cases} \frac{r^I(3 - 6k_m + 4k_m^3) + 3r^D}{6(1 - k_m)} & \text{for } r^I \leq 3r^D \\ \frac{4r^I(1 - k_m)^3 + 3r^D}{6(1 - k_m)} & \text{for } r^I > 3r^D . \end{cases} \tag{A12}$$

It is easy to check that when the merged banks set k_m at the level that is optimal for competitors the financing costs of the merged banks are always lower than the ones of the competitors. *A fortiori*, this must be true when they set k_m to minimize their financial costs. \square

PROOF OF PROPOSITION 3: The merged banks choose r_1^L and r_2^L to maximize equation (11) while competitors choose r_i^L to maximize equation (8) where the subscript i is now c . Define from the financing costs in Corollary 2 (equations (A11) and (A12)) the total marginal costs of the competitors and the merged banks as

$$c_c = c + \frac{r^I(1 - k_c)^2 + r^D}{2(1 - k_c)} \tag{A13}$$

and

$$c_m = \begin{cases} \beta c + \frac{r^I(3 - 6k_m + 4k_m^3) + 3r^D}{6(1 - k_m)} & \text{for } r^I \leq 3r^D \\ \beta c + \frac{4r^I(1 - k_m)^3 + 3r^D}{6(1 - k_m)} & \text{for } r^I > 3r^D , \end{cases} \tag{A14}$$

respectively. Using the expressions for k_m and k_c in equations (A8) and (A9), those for c_c and c_m in equations (A13) and (A14), $D_m = R_m + L_1 + L_2$ and $D_c = R_c + L_c$, we can write the expected profits for the merged banks and competitors when reserves are chosen optimally as

$$\begin{aligned} \Pi_m &= r_1^L L_1 + r_2^L L_2 - c_m(L_1 + L_2) \\ \Pi_c &= (r_1^L - c_c)L_c, \end{aligned}$$

where

$$\begin{aligned} L_m = L_1 + L_2 &= \left[l - \gamma \left(r_1^L - \frac{1}{N} \sum_{j=1}^N r_j^L \right) \right] \\ &+ \left[L - \gamma \left(r_2^L - \frac{1}{N} \sum_{j=1}^N r_j^L \right) \right], \end{aligned} \tag{A15}$$

and L_c is given by equation (2). The first-order conditions are then given by

$$\frac{\partial \Pi_m}{\partial r_h^L} = L_h + (r_1^L - c_m) \frac{\partial L_1}{\partial r_h^L} + (r_2^L - c_m) \frac{\partial L_2}{\partial r_h^L} = 0 \text{ for } h = 1, 2 \tag{A16}$$

$$\frac{\partial \Pi_c}{\partial r_i^L} = L_c + (r_i^L - c_c) \frac{\partial L_c}{\partial r_i^L} = 0 \text{ for } i = 3 \dots N. \quad (\text{A17})$$

We look at the post-merger equilibrium where $r_1^L = r_2^L = r_m^L$ and $r_i^L = r_c^L$. Substituting equation (A15) in equations (A16) and (2) in equation (A17), we obtain the best response functions as

$$r_m^L = \frac{l}{2\gamma \left(\frac{N-2}{N} \right)} + \frac{c_m}{2} + \frac{r_c^L}{2}. \quad (\text{A18})$$

$$r_c^L = \frac{l}{\gamma \left(\frac{N+1}{N} \right)} + \left(\frac{N-1}{N+1} \right) c_c + \frac{2}{N+1} r_m^L. \quad (\text{A19})$$

Solving equations (A18) and (A19) gives the post-merger equilibrium loan rates r_m^L and r_c^L . Substituting r_m^L and r_c^L , respectively in equation (A15) and in equation (2) gives the equilibrium L_m and L_c . Analogously, we derive D_m and D_c .

PROOF OF COROLLARY 3: Using equation (A4), we can express the liquidity risk for the merged banks as

$$\phi_m = \Pr(x_m > R_m) = \begin{cases} 1 - \int_0^{R_m} \frac{4x_m}{D_m^2} dx_m & \text{for } r^I \leq 3r^D \\ \int_{R_m}^{D_m} \frac{4(D_m - x_m)}{D_m^2} dx_m & \text{for } r^I > 3r^D. \end{cases}$$

Solving the integrals, we obtain $\phi_m = 1 - 2(R_m^2/D_m^2)$ for $r^I \leq 3r^D$ and $2 - 4(R_m/D_m) + 2(R_m^2/D_m^2)$ for $r^I > 3r^D$. Substituting $k_m = R_m/D_m$ implies

$$\phi_m = \begin{cases} 1 - 2k_m^2 & \text{for } r^I \leq 3r^D \\ 2(1 - k_m)^2 & \text{for } r^I > 3r^D. \end{cases}$$

Substituting k_m as in equation (A8), we can express the merged banks' resiliency as

$$1 - \phi_m = \begin{cases} 2[z(r^I, r^D)]^2 & \text{for } r^I \leq 3r^D \\ 1 - 2 \left(\sqrt[3]{\frac{3r^D}{8r^I}} \right)^2 & \text{for } r^I > 3r^D. \end{cases}$$

Similarly, from Corollary 1 we can write a bank's individual resiliency in the status quo as $1 - \phi_{sq} = k_{sq} = 1 - \sqrt{r^D/r^I}$. Plotting these expressions as a function of the ratio r^I/r^D , one immediately sees that $1 - \phi_m > 1 - \phi_{sq}$ always holds, so that $\phi_m < \phi_{sq}$. The plot is available from the authors upon request. \square

PROOF OF COROLLARY 4: Using equation (A4), we can express the expected liquidity needs for the merged banks as

$$\omega_m = \begin{cases} \int_{R_m}^{\frac{D_m}{2}} (x_m - R_m) \frac{4x_m}{D_m^2} dx_m \\ + \int_{\frac{D_m}{2}}^{D_m} (x_m - R_m) \frac{4(D_m - x_m)}{D_m^2} dx_m & \text{for } r^I \leq 3r^D \\ \int_{\frac{D_m}{2}}^{D_m} (x_m - R_m) \frac{4(D_m - x_m)}{D_m^2} dx_m & \text{for } r^I > 3r^D. \end{cases}$$

Solving the integrals, we obtain $\omega_m = (D_m/2) - R_m + (2/3)(R_m^3/D_m^2)$ for $r^I \leq 3r^D$ and $(2/3)[(D_m - R_m)^3/D_m^2]$ for $r^I > 3r^D$. Substituting $k_m = R_m/D_m$ we obtain

$$\omega_m = \begin{cases} \left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3\right) D_m & \text{for } r^I \leq 3r^D \\ \frac{2}{3}(1 - k_m)^3 D_m & \text{for } r^I > 3r^D. \end{cases}$$

To compare ω_m with $2\omega_{sq}$, we substitute equation (A8) in the above expression for ω_m and equation (A9) in the expression for ω_{sq} as in Corollary 1. We obtain:

$$\omega_m - 2\omega_{sq} = \begin{cases} \left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3\right) D_m - (1 - k_{sq})^2 D_{sq} & \text{for } r^I \leq 3r^D \\ \frac{r^D}{r^I} \left(\frac{D_m}{4} - D_{sq}\right) & \text{for } r^I > 3r^D. \end{cases}$$

For $r^I > 3r^D$ it is immediate to see that $\omega_m - 2\omega_{sq} < 0$ if $D_m/D_{sq} < 4$. For $r^I \leq 3r^D$, $\omega_m - 2\omega_{sq}$ can be rearranged as

$$\omega_m - 2\omega_{sq} = (1 - k_{sq})^2 D_{sq} \left[\frac{\left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3\right) D_m}{(1 - k_{sq})^2 D_{sq}} - 1 \right].$$

Suppose for a moment $k_m = k_{sq}$ and $D_m = 2D_{sq}$. Then, the expression simplifies to $k_{sq}^2 D_{sq} [(4/3)k_{sq} - 1]$, which is negative because $k_{sq} < 1/2$. To see that this holds also for $k_m > k_{sq}$, we use equation (A9) and rewrite $\omega_m - 2\omega_{sq}$ as

$$\omega_m - 2\omega_{sq} = \frac{r^D}{r^I} D_{sq} \left[\frac{r^I}{r^D} \left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3\right) \frac{D_m}{D_{sq}} - 1 \right].$$

Denote now $A = [(1/2) - k_m + (2/3)k_m^3]$. Since A is decreasing in k_m and $k_m > k_{sq}$ for $r^I \leq 3r^D$, it follows $\omega_m - 2\omega_{sq} < 0$ when $D_m = 2D_{sq}$. The same holds for

$D_m/D_{sq} < 2$. By plotting the expression $[(r^I/r^D)A(D_m/D_{sq}) - 1]$ for $D_m/D_{sq} > 2$ and $r^I/r^D \in (1, 3]$, one sees that there is a level $h \in (2, 4)$ of the ratio D_m/D_{sq} such that $\omega_m \leq 2\omega_{sq}$ if $D_m/D_{sq} \leq h$, and $\omega_m > 2\omega_{sq}$ otherwise. The plot is available from the authors upon request. \square

PROOF OF PROPOSITION 4: This proof is a generalization of that of Lemma 1. Let D_{tot} denote the total deposits $ND_{sq} = D_m + (N - 2)D_c$, and let R_{tot} denote the total reserves $NR_{sq} = R_m + (N - 2)R_c$. Applying the general formula for the distribution of a weighted sum of uniformly distributed random variables in Bradley and Gupta (2002) to our model, we obtain the density functions of the aggregate liquidity demands in the status quo $f_{sq}(X_{sq})$ and after the merger $f_m(X_m)$ as

$$f_{sq}(X_{sq}) = \frac{1}{(N - 1)!(D_{sq})^N} \sum_{i=0}^N [(-1)^i \binom{N}{i} (X_{sq} - iD_{sq})_+^{N-1}],$$

$$f_m(X_m) = \frac{\sum_{i=1}^{N-2} [(-1)^i \binom{N-2}{i-1} (X_m - D_m - (i - 1)D_c)_+^{N-2} + \binom{N-2}{i} (X_m - iD_c)_+^{N-2}]}{(N - 2)!D_m(D_c)^{N-2}}.$$

The two density functions are plotted in Figure 3. The density $f_{sq}(X_{sq})$ is more concentrated around the mean than $f_m(X_m)$. To verify that this is always the case, we compare the variances of X_{sq} and X_m , which are given by

$$\text{Var}(X_{sq}) = \sum_{i=1}^N D_{sq}^2 \text{Var}(\delta_i),$$

$$\text{Var}(X_m) = \frac{D_m^2}{4} \text{Var}(\delta_1) + \frac{D_m^2}{4} \text{Var}(\delta_m) + \sum_{i=3}^N D_c^2 \text{Var}(\delta_i)$$

$$= \text{Var}(\delta_i) \left[\frac{D_m^2}{2} + \sum_{i=3}^N D_c^2 \right],$$

because $\text{Var}(\delta_1) = \text{Var}(\delta_2) = \text{Var}(\delta_i)$. Since $D_m + \sum_{i=3}^N D_c = \sum_{i=1}^N D_{sq}$, one obtains $[\sum_{i=1}^2 (D_m^2/4) + \sum_{i=3}^N D_c^2] > \sum_{i=1}^N D_{sq}^2$ by Lagrangian maximization. Hence, it is always $\text{Var}(X_m) > \text{Var}(X_{sq})$. Since $f(X_{sq})$ and $f(X_m)$ are well behaved (they approach a normal distribution), they intersect only in two points.¹¹ This, along with the symmetry of the two density functions around the same mean $E[X_m] = E[X_{sq}] = D_{tot}/2$ and $\text{Var}(X_m) > \text{Var}(X_{sq})$, implies

11. A formal proof that this is the case is in Manzanares (2002).

$$\Phi_{sq} = \Pr(X_{sq} > R_{tot}) > \Phi_m = \Pr(X_m > R_{tot}) \text{ for any } R_{tot} < D_{tot}/2,$$

and vice versa for $R_{tot} > D_{tot}/2$. Using Proposition 1, $R_{tot} = NR_{sq}$, and equation (1), we obtain that $R_{tot} < D_{tot}/2$ if $r^I/r^D < 4 \equiv \sigma$. The first statement follows.

Using the definition in equation (7), we have

$$\begin{aligned} \Omega_m - \Omega_{sq} &= \int_{R_{tot}}^{D_{tot}} (X_m - R_{tot}) f_m(X_m) d(X_m) - \int_{R_{tot}}^{D_{tot}} (X_{sq} - R_{tot}) f_{sq}(X_{sq}) d(X_{sq}) \\ &= \int_{R_{tot}}^{D_{tot}} X_m f_m(X_m) d(X_m) - \int_{R_{tot}}^{D_{tot}} X_{sq} f_{sq}(X_{sq}) d(X_{sq}) \\ &\quad - R_{tot}(1 - F_m(R_{tot})) + R_{tot}(1 - F_{sq}(R_{tot})). \end{aligned}$$

Deriving it with respect to R_{tot} gives

$$\begin{aligned} \frac{d(\Omega_m - \Omega_{sq})}{dR_{tot}} &= -R_{tot} f_m(R_{tot}) + R_{tot} f_{sq}(R_{tot}) - (1 - F_m(R_{tot})) \\ &\quad + R_{tot} f_m(R_{tot}) + (1 - F_{sq}(R_{tot})) - R_{tot} f_{sq}(R_{tot}) \\ &= F_m(R_{tot}) - F_{sq}(R_{tot}). \end{aligned}$$

As shown earlier, $F_m(R_{tot}) - F_{sq}(R_{tot}) > 0$ for $R_{tot} < D_{tot}/2$ and $F_m(R_{tot}) - F_{sq}(R_{tot}) < 0$ for $R_{tot} > D_{tot}/2$. Also, $F_m(0) = F_{sq}(0) = 0$ and $F_m(R_{tot}) = F_{sq}(R_{tot}) = 0$. This implies $\Omega_m - \Omega_{sq} > 0$ for all $R_{tot} \in [0, D_{tot}]$. The second statement follows. \square

PROOF OF LEMMA 3: Suppose first $r^I/r^D < \rho$. In this range, the aggregate reserve/deposit ratio in the status quo (which coincides with the individual banks' deposit ratio) is smaller than the one after merger; that is,

$$k_{sq} = \frac{R_{sq}}{D_{sq}} = \frac{\sum_{i=1}^N R_{sq}}{ND_{sq}} < K_m$$

because $k_m > k_c = k_{sq}$. Consider now the aggregate liquidity risk. When $D_m = 2D_c$, this is given by

$$\Phi_{sq} = \text{prob} \left(\sum_{i=1}^N \delta_i D_{sq} > \sum_{i=1}^N R_{sq} \right) = \text{prob}(X' < k_{sq})$$

in the status quo, and by

$$\Phi_m = \text{prob} \left(\sum_{i=1}^N \delta_i D_c > R_m + \sum_{i=3}^N R_c \right) = \text{prob}(X' < K_m),$$

after the merger, where $X' = \sum_{i=1}^N \delta_i / N$. Since $K_m > k_{sq}$, it follows $\Phi_m < \Phi_{sq}$.

We can then express the expected aggregate liquidity needs in the status quo as

$$\begin{aligned} \Omega_{sq} &= \int_{k_{sq}ND_{sq}}^{ND_{sq}} (X_{sq} - k_{sq}ND_{sq})f(X_{sq}) d(X_{sq}) \\ &= ND_{sq} \int_{k_{sq}}^1 (X' - k_{sq})f(X') d(X'). \end{aligned}$$

Applying the same logic, the post-merger expected aggregate liquidity needs are

$$\begin{aligned} \Omega_m &= ND_c \int_{K_m}^1 (X' - K_m)f(X') d(X') \\ &= ND_{sq}(1 + (K_m - k_{sq})) \int_{K_m}^1 (X' - K_m)f(X') d(X'), \end{aligned}$$

where we have used $D_m = 2D_c$ and $D_m + (N - 2)D_c = ND_c = ND_{sq} + (K_m - k_{sq})ND_{sq}$. Given $K_m > k_{sq}$, we can write the expected aggregate liquidity needs as

$$\begin{aligned} \Omega_{sq} &= ND_{sq} \left[\int_{K_m}^1 (X' - k_{sq})f(X') d(X') + \int_{k_{sq}}^{K_m} (X' - k_{sq})f(X') d(X') \right] \\ &= ND_{sq} \left[\int_{K_m}^1 (X' - K_m)f(X') d(X') + (K_m - k_{sq}) \int_{K_m}^1 f(X') d(X') \right. \\ &\quad \left. + \int_{k_{sq}}^{K_m} (X' - K_m)f(X') d(X'), \right] \end{aligned}$$

and, after rearranging and simplifying, we have

$$\Omega_m - \Omega_{sq} = ND_{sq} \left[\begin{aligned} &(K_m - k_{sq}) \int_{K_m}^1 (X' - K_m - 1)f(X') d(X') \\ &- \int_{k_{sq}}^{K_m} (X'_{sq} - K_m)f(X') d(X') \end{aligned} \right] < 0$$

because $(X' - K_m - 1) < 0$. Analogous steps can be followed for the case $r^I/r^D > \rho$. □

PROOF OF PROPOSITION 5: Proposition 4 implies that if $k_m = k_{sq}$, then $\Phi_m > \Phi_{sq}$ and $\Omega_m > \Omega_{sq}$ for any $r^I/r^D > \rho$. A fortiori, this must be true in equilibrium where $k_m < k_{sq}$ (Φ_m and Ω_m are decreasing in K_m , which falls with k_m). □

PROOF OF PROPOSITION 6

Statement 1. From the proof of Proposition 4, $K_m = k_{sq}$ implies $\Phi_m = \Phi_{sq}$ when $r^I/r^D = \sigma$, and $\Phi_m < \Phi_{sq}$ when $r^I/r^D < \sigma$. Since $K_m > k_{sq}$ in the range $r^I/r^D < \rho$,

it is $\Phi_m < \Phi_{sq}$ when $r^I/r^D = \sigma$. The strict inequality and continuity imply that there must exist a neighborhood where $r^I/r^D > \sigma$ and $\Phi_m < \Phi_{sq}$. For $r^I/r^D > \rho$, $\Phi_m > \Phi_{sq}$ (from Proposition 5); hence, there must exist a critical level $g \in (\sigma, \rho)$ (with $\sigma < \rho$ from the proofs of Propositions 2 and 4) such that as $\Phi_m < \Phi_{sq}$ if $r^I/r^D < g$, and $\Phi_m > \Phi_{sq}$ otherwise. The first statement follows.

Statement 2. From Proposition 2, $k_m = k_{sq}$ for $r^I/r^D = 1$ and $r^I/r^D = \rho$, and $k_m > k_{sq}$ for $1 < r^I/r^D < \rho$. This induces the same relation between K_m and k_{sq} , so that $K_m - k_{sq}$ is first increasing and then decreasing in the interval $r^I/r^D \in (1, \rho)$. By Proposition 4, when $D_m \neq 2D_c$ there is a neighborhood of $r^I/r^D = 1$ where $\Omega_m - \Omega_{sq} > 0$. Also, when $r^I/r^D = \rho$ and $D_m \neq 2D_c$, $\Omega_m > \Omega_{sq}$. When $r^I/r^D = 1$, it is always $\Omega_m = \Omega_{sq} = D_{tot}/2$. From Lemma 3, when $D_m = 2D_c$ it is $\Omega_m - \Omega_{sq} < 0$ for all $r^I/r^D \in (1, \rho)$ and $\Omega_m = \Omega_{sq}$ when $r^I/r^D = \rho$. By continuity, if one fixes a sufficiently small level of asymmetry in the deposit bases across banks ($D_m - 2D_c$ sufficiently small), then $\Omega_m - \Omega_{sq} > 0$ in an immediate neighborhood of $r^I/r^D = 1$. Given that $K_m - k_{sq}$ is increasing around $r^I/r^D = 1$, there will be a higher ratio r^I/r^D , named g , such that if the merger generates that asymmetry when $r^I/r^D = g$, then $\Omega_m - \Omega_{sq} = 0$ and $\Omega_m - \Omega_{sq} < 0$ in the immediate right neighborhood. Again by continuity, $\Omega_m - \Omega_{sq} > 0$ in an immediate neighborhood of $r^I/r^D = \rho$. Given that $K_m - k_{sq}$ is decreasing around $r^I/r^D = \rho$, there will be a smaller ratio r^I/r^D , named \bar{g} , such that, when $r^I/r^D = \bar{g}$, then $\Omega_m - \Omega_{sq} = 0$ and $\Omega_m - \Omega_{sq} < 0$ in the immediate left neighborhood. The second statement follows. \square

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