Relational Efficient Property Rights*

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Abstract

We propose a simple and general framework for comparing different ownership structures with respect to creating appropriate incentives for cooperative behavior (efficient investment) in long-run business relationships, and introduce the notion *relational efficiency* based on Abreu's (1986, 1988) definition of optimal punishment. We identify relational efficient ownership structures and find that the short term efficient ownership structure in the tradition of Grossman, Hart and Moore is generally not relational efficient, although it tends to be *constrained* efficient, the constraint being the use of grim trigger strategies. We generalize models by Garvey (1995) and Halonen (2002) and reconsider that of Baker, Gibbons and Murphy (2001, 2002), confirming some of their results but not others.

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1 Introduction

The property rights theory of the firm, as formally developed by Grossmann and Hart (1986) and Hart and Moore (1990), has probably been, toghether with Williamson's (1975, 1985) and Klein, Crawford and Alchian's (1978) transaction costs theory, the most important advancement in the understanding of the determinants of the boundary of the firm since Coase's (1937) seminal contribution.[1]

The property rights theory, however, focuses on individual, isolated transactions regulated by explicit, incomplete contracts.[2] Simon (1951), Macaulay (1963), Klein and Leffler (1981), and Telser (1981), among others, have stressed instead that the best part of economic transactions are not isolated exchanges, but rather episodes of a history of repeated exchanges; and that one can hardly build a complete theory of the firm without taking into account the informal agreements that complement explicit contracts in governing business relationships.

Relational contracts, flexible self-enforcing informal arrangements among employees, are essential to "have things running smoothly" within organizations. An obvious sign of this is that even in most developed countries with detailed explicit employment contracts and efficient law enforcement, "working to rule" – i.e. following literally what is prescribed by the explicit contract – is among the tougher traditional weapons in the hands of employees when bargaining for higher wages.[3] Relational contracts are also crucial between organizations, in particular for the governance of specific supply relations and joint ventures: the most complex and important aspects of these transactions are typically the harder to contract upon explicitely, and still long term cooperation on such crucial aspects is normally achieved between producers and suppliers in complex industries.[4] Even in large, infrequent public procurements like Public-Private Partnerships, the best case one can imagine for explicit contracts to be crucial (at least of advanced countries with sophisticated legal systems: infrequent contracts and public law restrict the use of reputational forces and non verifiable/auditable information), these are in fact useful in so far as they facilitate cooperative behavior among transacting parties, and

[1]Whinston (2003) nicely clarifies the differences among these two theories, focussing on their distinct (and somewhat hard to test) empirical implications and stressing the difficulty of testing those of the property right theory.

[2]See Hart (2005) for an overview. We do not enter the debate on why explicit contracts are incomplete taking for granted that they generally are. The interested reader may for example want to look at the special issue in RES 66 (1999). Independent of complexity considerations, the mere presence of (typically very high) costs of court enforcement justify the limited use of detailed explicit contracts often observed in reality. Battigalli and Maggi (2002) show that even extremely small costs of writing contracts leads to a substantial degree of optimal contract incompleteness.

[3]Lazzarini et al. (2004) provide nice experimental evidence how explicit and implicit contracts complement each other in governing moral hazard in economic transactions.

[4]Fehr, Brown, and Falk (2004) provide striking experimental evidence of the overwhelming importance of relational contracts when non-contractable aspects are present in a transaction.

they may become a burden to the parties otherwise.[5]

Because explicit contracts enforced by courts act mainly as a constraining framework within which collaborative business relationships develop, understanding the interaction between (explicit) property rights on assets instrumental for production and the relational agreements among parties that complete them is crucial to the theory of organizations. The aim of this paper is to provide a simple and general framework for analyzing and comparing different ownership structures (and potentially other explicit contracts) with respect to their effects on parties' ability to sustain the relational contracts necessary to achieve productive efficiency in long-run business partnership.

The formal theory of implicit/relational contracts is well developed since Bull (1987) and MacLeod and Malcomson (1989), and has been considerably extended by Levin (2003). Authors like Garvey (1995), Halonen (2002), and Baker, Gibbons and Murphy (2001, 2002) have begun to analyze formally how asset ownership and relational contracts interact within dynamic models of ongoing organizations.[6] A message common to all these papers is that in a dynamic framework where relational contracts are important, ownership still matters (when self-enforcing constraints bind), but for somewhat different reasons than in the Hart and Moore (1990) framework, so that the optimal ownership structure may differ from the one identified by Hart and Moore.[7]

Apart from this common message, the results of these contributions differ substantially and are not easy to compare since they differ in their specifications, focus and additional assumptions.[8] The literature restricts focus only to special strategies, "grim trigger" or Nash reversion (play cooperatively; if a defection takes place, revert for ever to the static equilibrium, with or without ownership renegotiation along the punishment path). These strategies are easy to understand, and therefore familiar in the applied literature, and, perhaps, used in

[5] Public-Private Partnerships are typically regulated by complex, detailed explicit contracts that have sometimes taken over a year to complete. Still, in evaluating problems and factors of success for the Private Finance Initiatives, the main form of PPPs in the UK (the leading county on PPP), the UK Treasury (2006) writes that "*A spirit of partnership espoused between the contracting parties is key for successful performance...*", that "*...there is a balance to be struck between partnering and contract management and enforcement...*", and that "*...informal agreements have often been developed by contract managers to make the management of changes easier.*".

[6] The work of Bernheim and Whinston (1998) and Rosenkrantz and Schmitz (2001) should also be mentioned, although their two-stage models fall somewhat short of describing a long-term partnership.

[7] The result is in the same spirit of Che and Yoo (2001), who showed that in a dynamic framework team-based incentives may be optimal where they are not in a static one (see also Spagnolo, 1999). A different message comes from Rajo (2003) and Rajo and Levin (2003), who show that concentrating residual claims on profit streams and control (discretion) tends to be relational efficient.

[8] Some contributions argue for example that short term and long term optimal ownership structure actually coincide. See for example Hart (2001), and Bragelien (2002) who extends Baker, Gibbons and Murphey's framework to the case of two investing parties

reality; but from a theoretical point of view, they are somewhat *ad hoc*. An exclusive focus on these strategies makes it hard to assess how robust or general the conclusions of these earlier models are.[9] Analogously, previous models restrict focus to small subsets of possible ownership structures, so that it is not clear whether a wider choice of ownership structures, as available in reality, would change the results.

In this paper we try to clarify the role of ownership in the ability to maintain a partnership by setting up a simple but general model, and by making full use of established results in the theory of repeated and dynamic games. One main distinguishing feature of our approach relative to previous work is the absence of restrictive assumptions on which strategies are played. An other important feature is that we do not restrict attention to the question "to buy or to make?" or the outsourcing/integration trade-off, but allow for arbitrary other forms of ownership. This includes joint ownership, but also the case where some assets are owned jointly and others individually. Finally, we always study both cases with and without renegotiation.

The main idea of this article is the concept of a *relational efficient* or *long-term-efficient ownership structure*. An ownership structure is called relational efficient if it supports efficient or co-operative behavior as subgame perfect equilibrium in a discounted repeated game for a maximal range of discount factors. We compare different constellations of property rights on assets with respect to their relative ”relational efficiency”, providing a full characterization of relational efficient ownership structures.[10]Our main findings are that, in general, relational efficient and "short term efficient" ownership structures (as identified by the property rights theory) do not coincide; and that in contrast to short term efficiency and to previous work on relational contracts and property rights, the possibility to renegotiate ownership after a break down of cooperation does not influence the relational efficient ownership structure.

These results confirm, generalize, and extend the message common to Halonen (2002), Garvey (1995)) and Baker, Gibbons and Murphy (2002), that ownership does matter in dynamic business relationships, but for somewhat different reasons than in the Hart and Moore's framework. However, we also find that Hart and Moore's short term efficient structure tends to be constrained relational efficient, the constraint being the use of grim trigger strategies, and we identify specific situations in which it is also fully relational efficient.

Our theory yields progress on the question how a business partnership should be organized in order to maximize its stability and success. Since patience of real world business partners or the probability of survival of the partnership may vary over time for exogenous reasons

[9] Moreover, grim trigger strategies are generally not optimal for the agents (in the sense of Abreu 1986, 1988), nor are they robust to ex post renegotiation (in the sense of Farrell and Maskin, 1989) or to trembles and mistakes (in the sense of Segerström, 1988).

[10] We also take into account the "constrained" form of relational efficiency considered in previous work, i.e. relational efficiency under the constraint that agents use grim trigger strategies.

and is typically not observed directly, our theory provides a logical link between observable characteristics as the *value of the partnership* and the allocation of ownership rights. The value of the partnership is defined as the discounted expected stream of joint returns generated by acting together co-operatively on top of the respective value generated in the next best individual use of assets under the respective ownership structure. Our main theorems 1 and 2 establish that in the relationally efficient ownership structure not only the incentives to misbehave are be minimized but also the punishment to non-cooperative behavior is maximized. Generally, to construct an institution with maximal punishment power against misbehavior one has to give up on the goal of short term second best efficiency. One very practical method to raise punishment power is the introduction of veto rights over certain assets. In many countries there are thresholds to minorities of shareholders above which these can exert "punishment power" in the form of veto rights with respect to certain types of decisions of the management. The power to constrain the choice set of a business partner with respect to certain assets establishes an ownership structure that decreases short term efficiency but raises punishment power and thereby relational efficiency in our framework. If increased relational efficiency is publicly observable and is priced into the companie's value by efficient markets critical transitions in the ownership structure should reflect this. This observation brings about various testable predictions of our theory:

(*i*) In cases, where a publicly listed companie's stakeholder aquires the status of veto-power over some of the management's decisions the value of the company should increase.

(*ii*) The increase of value should be smaller if a stakeholder raises his stake but does not aquire similar veto-power.

(*iii*) We expect to observe significant increases of stock market prices at times where legislation decreases the thresholds for blocking minorities or increases the veto-power of relevant business parties.

(*iv*) The formation of joint ventures or joint ownership of assets should be observed more frequently in long-run business partnerships as compared to individual, isolated transactions regulated by explicit, incomplete contracts.

Note that only the first of these four predictions is also compatible with explanations in the tradition of Shleifer and Vishny (1986), where large shareholders exert positive externalities by being able to monitor manager's decisions independently of their "repeated game punishment power".

As for other, more specific theoretical results, we reformulate Garvey's (1995) and Halonen's (2002) models as sub-specifications of our model and re-examine them without assumptions on strategies. Halonen's result that joint ownership may be optimal in a dynamic environment is confirmed and generalized to the case when renegotiation is possible.[11] Garvey's result

[11] Halonen (2002) has been criticized for assuming away the possibility to (costlessly) renegotiate ownership

that relational contracting requires more symmetric ownership structures is instead weakened, as with optimal transfers and strategies in his model ownership becomes irrelevant. In an appendix we also reconsider Baker, Gibbons and Murphy's richer model without assumptions on strategies, confirming and extending the main result in Baker, Gibbons and Murphy (2002) on the importance of relational contracts for the decision whether to integrate. Baker Gibbons and Murphy's (2001) more specific result on the impossibility to "bring the market inside the firm" does not generalize instead, once assumptions about strategies are removed.

The reminder of paper unfolds as follows. Section 2, describes the investment stage game. Section 3 describes the dynamic/repeated game, defines long term (relational) efficiency and states the main results. Section 4 applies the general framework to the model specifications of Garvey (1995) and Halonen (2002). Section 5 concludes. All proofs are in the Appendix.

2 Investment stage game

Two parties $i = 1, 2$ play the following investment stage game. In the first substage both parties decide simultaneously on a non-contractible costly action $e_i \in E_i$, with cost $C_i(e_i)$, that can be interpreted as effort or non-contractible investment. In the second substage agents bargain over the jointly created "cake" $Q(e_1, e_2) \geq 0$. The size of this cake depends on both actions. Let

$$e = (e_1, e_2) \in E := E_1 \times E_2$$

denote an action profile. We assume for simplicity that there exists a unique action profile $e^c = (e_1^c, e_2^c) \in E$ that maximizes the size of the joint surplus given as

$$S(e) = Q(e) - C_1(e_1) - C_2(e_2)$$

and call $e_i^c \in E_i$ the "cooperative" or "first-best" action of agent i. In the tradition of Grossmann-Hart-Moore agents' actions not only determine the joint surplus but also their own bargaining positions. To model this, introduce the threat point or disagreement (or status quo) payoffs

$$(P_1(e,\omega), P_2(e,\omega)) \in \left\{(x_1, x_2) \in R^2 \,|x_1 + x_2 \leq Q(e)\right\}.$$

Agent i's disagreement payoff $P_i(e,\omega)$ depends on parameter $\omega \in \Omega$ – interpreted as "ownership structure" – and on action profile e. Our interpretation is that agent i obtains $P_i(e,\omega)$ by using his control rights over assets for the next best alternative outside the relationship[12].

after cooperation breaks down, a possibility that in her framework destroys her result (see e.g. Bragelien (2002)). We show here that this objection, although justified in itself, does not change the results when agents are free to choose optimally the strategies by which to support efficient investments.

[12] We allow explicitly for the case that the disagreement payoff depends on both parties' actions. For example some assets may be worthless without outside expertise.

Call $Q(e) - P_1(e,\omega) - P_2(e,\omega)$ the *value* of the relationship or the "net-cake". We assume that the value is positive $Q(e) - P_1(e,\omega) - P_2(e,\omega) \geq 0$ and stick to the frequently used assumption that the "net-cake" $Q(e) - P_1(e,\omega) - P_2(e,\omega)$ is split equally within the relationship, i.e. parties agree on the Nash-Bargaining-Solution (NBS) yielding payoff

$$u_i(e,\omega) = \frac{1}{2}\left[Q(e) + P_i(e,\omega) - P_{-i}(e,\omega)\right] - C_i(e_i) \tag{1}$$

to agent i.[13] Note that the joint surplus $S(e) = u_1(e,\omega) + u_2(e,\omega)$ and in particular the first best joint surplus

$$S^* \equiv u_1(e^c,\omega) + u_2(e^c,\omega) = Q(e_1^c, e_2^c) - C_1(e_1^c) - C_2(e_2^c) \tag{2}$$

do not depend on ownership ω. For any ownership structure $\omega \in \Omega$ the payoffs $u_i(e,\omega)$ define a game with simultaneous choice of actions in the first substage denoted by $\Gamma(\omega)$ and called subsequently the "reduced form stage game".

The salient topic of the literature on ownership rights builds on the observation that maximization of individual payoffs $u_i(e,\omega)$ and of joint surplus $S(e) = u_1(e,\omega) + u_2(e,\omega)$ create different incentives. In order to relate our results to this literature we focus on structures where it is not in both agents' individual short term interest to act cooperatively.

Definition 1 *A family of stage games $\{\Gamma(\omega)\}_{\omega\in\Omega}$ is called a "holdup structure" iff*

(i) for every $\omega \in \Omega$ each agent $i = 1,2$ has a unique best response strategy denoted by $e_i^b(e_{-i},\omega)$ where $e_i^b(e_{-i}^c,\omega) \neq e_i^c$ for all $\omega \in \Omega$ and

(ii) $\Gamma(\omega)$ has a unique (pure strategy) Nash equilibrium called "holdup equilibrium" denoted by $e^d(\omega) = (e_1^d(\omega), e_2^d(\omega))$. This implies $e_i^d(\omega) = e_i^b(e_{-i}^d,\omega) \neq e_i^c$.

To keep notation suggestive and simple we introduce the following shortcut variables for

[13] It is well known that allowing for "outside options" may change the conclusions of the Grossmann-Hart-Moore approach (De Meza and Lookwood, 1998; Chiu, 1998). Since we are mainly interested in the long term versus short term comparison we follow the more simple traditional route with regard to the formulation of the stage game.

$i = 1, 2$

$$\begin{aligned}
c_i(\omega) &= u_i(e^c, \omega) \text{ for "Cooperation payoff"}, && (3a)\\
d_i(\omega) &= u_i\left(e^d(\omega), \omega\right) \text{ for "Defection payoff" = holdup equilibrium payoff}, && (3b)\\
b_i(\omega) &= u_i\left(\left(e_i^b\left(e_{-i}^c, \omega\right), e_{-i}^c\right), \omega\right) \text{ for "Betray payoff"}, && (3c)\\
a_i(\omega) &= u_i\left(\left(e_i^c(\omega), e_{-i}^b\left(e_i^c, \omega\right)\right), \omega\right) \text{ for "Affected payoff"}, && (3d)\\
S^* &= c_1(\omega) + c_2(\omega) \text{ joint first best surplus}, && (3e)\\
D(\omega) &= d_1(\omega) + d_2(\omega) \text{ joint holdup equilibrium payoffs}, && (3f)\\
B(\omega) &= b_1(\omega) + b_2(\omega) \text{ joint betray payoffs}. && (3g)\\
HP(\omega) &= S^* - D(\omega) \text{ the } \textit{size} \text{ of the holdup problem quantified by its efficiency loss}. && (3h)
\end{aligned}$$

Capital letters D, B stand for aggregates. In contrast to the "first best" cooperative action profile e^c the holdup equilibrium $e^d(\omega)$ depends on ownership structure ω. Therefore, we can compare different ownership structures with respect to the sum of the generated (transferable) equilibrium utilities.

Definition 2 *Call $\omega^* \in \Omega$ "short term efficient ownership structure" if it maximizes the sum of equilibrium payoffs or minimizes the size of the holdup problem*

$$\begin{aligned}
D(\omega^*) &\geq D(\omega) \quad \forall \omega \in \Omega \Leftrightarrow && (3d)\\
HP(\omega^*) &\leq HP(\omega) \quad \forall \omega \in \Omega. && (3e)
\end{aligned}$$

The corresponding set of short term efficient ownership structures is denoted by Ω^*. Further, denote by $d_i^*(\omega) = u_i\left(e^d(\omega^*), \omega^*\right)$ the short term efficient holdup equilibrium payoff to party i and by $D^* = d_1^*(\omega) + d_2^*(\omega)$ the joint short term efficient holdup equilibrium payoffs.

Parties who recognize that cooperation is not sustainable have an incentive to renegotiate ownership whenever the initial ownership structure ω was short-term inefficient and the cost of renegotiation is sufficiently small. If the total cost of renegotiating/reallocating ownership is $z \geq 0$, agents' payoff increases by

$$T = \frac{1}{2}\left(D^* - D(\omega) - z\right)$$

if agents again apply Nash bargaining (split the pie).

3 Relational efficiency

From here our understanding of a *relationship* is that parties play repeatedly (and with positive probability) the investment stage game described in the previous section. We assume perfect monitoring, i.e. at the beginning of each stage game both players can see and remember the entire back history of the game. In the language of contract theory, we assume that investment decisions e_i are observable but not verifiable. At the beginning of each period parties can choose the ownership structure before playing the investment game. To support the efficient level of investment on the equilibrium path of this dynamic game – to maximize joint payoffs or the *pie* – agents may need to split the pie in a different way than specified by the payoffs of $\Gamma(\omega)$. We assume that to optimally adjust the shares of the pie, agents can choose a monetary transfer $\theta_1 = \theta = -\theta_2$, so that $u_i(e^c, \omega) + \theta_i$ goes to agent i if both agents cooperate. From here we call θ the *profit sharing rule* or simply *transfer*.[14]

The generic stage after history h of the dynamic game has the following three-step structure

1	2	3
ownership structure ω^h is chosen	$\Gamma(\omega^h)$ is played (simultaneous investments)	θ^h is paid (profits are split)

Call $\Gamma(\delta, \omega^h, \theta^h)$ the resulting dynamic game with joint discount factor δ. A strategy profile of $\Gamma(\delta, \omega^h, \theta^h)$ is denoted by $s = (s_1, s_2) \in S = S_1 \times S_2$.

To keep the model tractable we do not explicitly model how parties reach an agreement over the ownership structure and the transfer payments. Step 1 of the stage game is assumed to be contractible and verifyable. However, an explicit contract regarding ownership cannot be made contingent on investment decisions since these are not contractible. Conversely the transfer specified by the profit sharing rule is interpreted as a voluntary payment that is made to make the relationship 'work smoothly'[15]. For the determination of strategic investment decisions we treat ownership and transfer as exogenously given. As a reduced form of a fully specified non-co-operative bargaining model we then determine ownership and profit sharing rule endogenously by efficiency. More precisely, we call a system of ownership structures and profit sharing rules (ω^h, θ^h) *renegotiation proof* if there exists no alternative system $(\omega^h, \theta^h)'$ which constitutes a Pareto improvement[16] in the continuation payoffs after any history h. The interpretation is that if after some history h parties could raise their joint payoffs by writing a

[14] In the background of this formulation are efficient equilibria of a model where parties can negotiate and are able to commit to the transfer payment path before playing the supergame. Within any continuation supergame this path is then exogenous. This motivates our language that agents can choose a transfer. The monetary transfer could in principle be contracted upon, but in equilibrium it will be part of the relational contract since by assumption investments are non-contractible.

[15] In our proof for the characterization of relational efficient ownership with renegotiation we make use of history dependant transfer payments that take the form of a fine. Relationships that do not work can be 'repaired' if a player who made a mistake pays a restitution transfer.

[16] Since utility is transferable this is equivalent to picking the dynamic game specified by (ω^h, θ^h) that maximizes the joint payoff of both parties.

new contract that specifies the change of ownership and transfer payments they would be able to do it even if co-operation did break down.

Now we set up our main concept, i.e. ownership structures that are most supportive for efficient, cooperative behavior in this dynamic game. In particular, we introduce the following efficiency criterium:

Definition 3 *Ownership structure* ω^h *and profit sharing rule* θ^h *are called "relational efficient" or "long term efficient" if they minimize the lower bound* $\underline{\delta}$ *on discount factors such that for all* $\delta \geq \underline{\delta}$ *there exists a subgame-perfect equilibrium* $s^* \in S$ *supporting indefinite cooperation with ownership structure* ω *on its equilibrium path for* $\Gamma\left(\delta, \omega^h, \theta^h\right)$*. Accordingly, call the respective equilibrium and the equilibrium strategies "relational efficient". The set of relational efficient ownership structures is denoted by* Ω^{**}*.*

Negotiations on ownership structure at the beginning of the dynamic game and in any other period may involve transfers between agents and take place at some cost $z \geq 0$. When $z = 0$, we would expect parties to renegotiate the ownership structure to one that is relational efficient at the beginning of each stage game.

To further motivate our definition of relational efficiency it is helpful to review briefly the assumptions that have been made in the previous literature.

Garvey (1995) and Baker, Gibbons and Murphy (2001, 2002) assume that agents support cooperation by grim-trigger strategies and that renegotiation cost is negligable or $z = 0$. In their framework, after a defection cooperation breaks down forever, but agents are still able to renegotiate ownership. At the beginning of the infinite punishment phase the ownership structure is renegotiated to the short-term efficient one, the gains from renegotiation being split according to symmetric Nash bargaining.

Halonen (2002) also assumes that agents support efficient investments through grim-trigger strategies. However, for most of her analysis she assumes renegotiation cost z to be large enough, so that ownership is not renegotiated when cooperation breaks down.[17]

We think that both assumptions on renegotiation $z = 0$ and z large can be defended and are worth being studied. However, imposing assumptions on strategies is restrictive. First, any assumption on strategies in our view is not consistent with non-contractibility of investment decisions. The very notion of non-contractibility is that there is no formal restriction on each party's actions. Rather, restrictions are endogeneous in the sense that strategies should be self-enforcing or, in the language of game theory, subgame-perfect equilibria of the repeated game.[18]

[17] Halonen (2002) also considers an example with renegotiation of ownership, but does not derive general results for that case.

[18] Moreover, since grim trigger strategies prescribe inefficient play forever after a defection, they are somewhat fragile with respect to trembles or mistakes. As convincingly argued by Segerström (1988), if agents make

Second, grim-trigger strategies exclude some potentially relevant equilibria since generally they are not optimal in the sense of Abreu (1986, 1988). Note that there may exist functioning relationships that only work because of the background-threat of the most severe punishment to a deviator that is available in a continuation equilibrium. In our context this implies that if parties would be restricted to less severe than maximal forms of punishment some relationships would not be viable altogether. On the other hand, for some results in the previously mentioned papers this objection is not a problem as long as the stage game is a Prisoner's Dilemma, since there grim-trigger strategies are optimal.

To sum up, our definiton of relational efficient ownership structures only relies on non-contractibility or self-enforcing behavior and does not contain assumptions on which strategies parties should or can play in a relationship. In what follows, we will consider both cases of z large and $z = 0$, but we will let agents free to choose strategies optimally, and we will pay attention to their ability to renegotiate strategies, besides ownership.

No renegotiation. By no renegotiation or $z \to \infty$ we mean that any change of ownership ω or transfer θ is prohibitively expensive. In this case the dynamic game $\Gamma\left(\delta, \omega^h, \theta^h\right)$ described above degenerates into a standard discounted repeated game $\Gamma(\delta, \omega, \theta)$. Abreu (1988) has shown that under mild regularity conditions for discounted repeated games there exist optimal punishments.[19] Therefore the optimal punishment continuation payoff for player i is well defined and is denoted by $U_i^{\min}(\omega)$. We use the notation $v_i(\omega) = (1-\delta) U_i^{\min}(\omega)$ and $V(\omega) = v_1(\omega) + v_2(\omega)$ to simplify exposition. The following theorem characterizes relational efficient transfers and ownership structures if parties cannot renegotiate.

Theorem 1 *Consider the discounted repeated investment game* $\Gamma(\delta, \omega, \theta)$. *An ownership structure* ω *is relational efficient if and only if*

$$\omega \in \Omega^{**} := \left\{\omega \,\middle|\, \underline{\delta}^{**}(\omega) \leq \underline{\delta}^{**}\left(\omega'\right) \;\; \forall \omega' \in \Omega\right\}$$

where

$$\underline{\delta}^{**}(\omega) = \frac{B(\omega) - S^*}{B(\omega) - V(\omega)}.$$

For any relational efficient ownership structure $\omega \in \Omega^{**}$ *the corresponding relational efficient transfer is given by*

$$\theta^{**}(\omega) = \frac{(b_1(\omega) - c_1(\omega))(b_2(\omega) - v_2(\omega)) - (b_2(\omega) - c_2(\omega))(b_1(\omega) - v_1(\omega))}{B(\omega) - V(\omega)}.$$

mistakes with positive probability, they will tend not to choose strategies by which cooperation breaks down forever after a mistake occurs.

[19] Action spaces E_i have to be compact topological spaces and payoff functions u_i must be continuous.

Since in general minimizing $\underline{\delta}^{**}(\omega) = \frac{B(\omega)-S^*}{B(\omega)-V(\omega)}$ and maximizing $D(\omega)$ leads to different results, an immediate implication of Theorem 1 is that the relational efficient ownership structure generally differs from the short term efficient ownership structure. Borrowing terminology from Compte et al. (2002), minimizing $\underline{\delta}^{**}(\omega)$ implies taking care of the "deviation concern" (minimizing short-run gains from deviating from the contract) *and* of the "punishment concern" (maximizing the sanctions against deviators). Essentially, the dynamic logic of these two incentives differs from the static logic behind Grossman, Hart and Moore's theory of maximizing payoffs in the one-shot Nash equilibrium of the stage game.

The following example, inspired by Halonen (2002), illuminates this discrepancy for a simple case.

Example 1 *Consider a stage game with a binary action space $E_i = \{e_i^c, e_i^d\}$ where $\Gamma(\omega)$ is a Prisoner's Dilemma. Let $Q(e^d) = P_i(e^d, \omega) + P_{-i}(e^d, \omega)$, i.e. the defective action profile e^d yields the same joint payoff as quitting the relationship. Hence, the minimal subgame perfect continuation payoff is given by playing defect forever, and $V(\omega) = D(\omega)$. Let further ownership structures be given as $\Omega = \{\omega_{SO}, \omega_{JO}\}$ interpreted as "single ownership" and "joint ownership" with similar aggregated betray payoffs $B(\omega_{SO}) = B(\omega_{JO})$ but a pronounced difference outside the relationship $D(\omega_{SO}) > D(\omega_{JO})$. In that case single ownership is short term efficient $\Omega^* = \{\omega_{SO}\}$ since it maximizes $D(\omega)$ whereas joint ownership is relational efficient $\Omega^{**} = \{\omega_{JO}\}$ since it minimizes $D(\omega)$ and therefore minimizes $\underline{\delta}^{**}(\omega)$. The intuition for this discrepancy is that dynamically motivated parties maximize the incentive to co-operate by organizing ownership such that not to co-operate is as unattractive as possible. In contrast, statically motivated parties recognize that co-operation cannot be achieved and organize ownership such that assets are most valuable in the absence of co-operation.*

Renegotiation. Considering the latter example of joint ownership it is intuitive that the possibility to renegotiate should affect the relational efficient ownership structure. Note that the logic of joint ownership is that co-operation works so well since non-co-operation is made very inefficient and thereby the threat of it is very effective. Hence, one would expect that if parties anticipate that payoffs can be improved more easily by renegotiating ownership after one party deviates and co-operation breaks down this would decrease the incentive to co-operate in the first place. The following theorem shows that this is not the case and again characterizes relational efficient ownership structures for this case.

Theorem 2 *Assume that parties can renegotiate ownership and transfers costlessly, i.e. $z = 0$. Consider the dynamic investment game $\Gamma(\delta, \omega^h, \theta^h)$. Then relational efficient transfer $\theta^{**}(\omega)$ and ownership structures Ω^{**} remain the same as in theorem 1.*

This theorem reinforces the claim of Theorem 1 showing that the set of relational efficient ownership structures Ω^{**} does not depend on the possibility to renegotiate. Rather, it only depends on gains from defection and optimal punishments, so it is again given by

$$\Omega^{**} := \left\{ \omega \left| \underline{\delta}^{**}(\omega) \leq \underline{\delta}^{**}\left(\omega'\right) \ \forall \omega' \in \Omega \right. \right\},$$

as stated in Theorem 1.

The intuition behind this result is as follows. In our contracting environment we allow parties to exchange monetary transfers. Levin (1998, 2003) observed (notably for repeated games with incomplete information) that when monetary transfers are possible, optimal punishments can take the simple, natural form of a "fine" levied against a player that defects, with play remaining on the equilibrium path in the continuation subgame. Any fine that results in a payoff strictly above the continuation payoff of the optimal punishment path is strictly preferred by a deviator compared to optimal punishment which can always be imposed without using fines. This last feature can be used to construct equilibria where no player ever has an incentive not to co-operate independent of the previous history. Any deviation is punished via a fine which in turn is backed by the physical optimal punishment established in theorem 1. However, strategies can be fine-tuned in a way that on each continuation path each player has no incentive to abandon equilibrium strategies by not co-operating or by not paying the fine. In addition, by keeping play on the Pareto efficient path, these strategies are as robust as feasible with respect to mistakes. This model is not a repeated game as the possibility to renegotiate and change ownership structure in each period makes it a fully dynamic game. Still, it turns out that natural, optimal asymmetric punishment strategies based on monetary fines can be constructed easily in our framework, with effects summarized by Theorem 2. Renegotiation of ownership never occurs along the punishment path since the original ownership structure and equilibrium are efficient, so that no renegotiation (of ownership, strategies or both) can bring about the Pareto improvement necessary to have both players' agreeing.

Grim-trigger strategies We do not recognize a good theoretical justification for imposing any assumptions on which specific strategies business partners play in relationships with *non-contractible* actions. As already mentioned, grim trigger strategies are only relevant for relational efficiency if they happen to be optimal punishment, which is not the case in general. Nevertheless, we think it is interesting to compare our approach with models using this assumption and to see to which extent removing the assumption affects the results. Therefore, in this section we investigate the consequences of imposing grim trigger strategies, i.e. that players switch forever to the unique static non-co-operative behavior after any defection. The ownership structure chosen for the relationship may not be efficient in the continuation paths where players do not co-operate, in which case renegotiation to the short term efficient ownership structure raises efficiency if it is not too costly. Garvey (1995) and Baker, Gibbons and Murphey (2001, 2002) assume that renegotiation to the short term efficient ownership structure is costless and always occurs after a defection.

The assumption to use less than maximal punishment and moreover the anticipated possibility to renegotiate ownership (at zero or low cost) to the Pareto-optimal level at each node down the game tree makes the efficient investment harder to sustain in equilibrium. In particular, renegotiation of ownership induces a lower bound on time-average payoffs during the punishment phase, namely the payoffs obtained from non-cooperative investment under the "short-term efficient" ownership structure ω^*. The incentive compatibility condition to cooperate supported by grim-trigger-strategies is given by

$$c_i(\omega) + \theta_i \geq (1-\delta)\, b_i(\omega) + \delta\,(d_i(\omega) + T),$$

where $c_i(\omega) + \theta_i$ goes to agent i if both agents cooperate. The new sharing rule θ_1 depends now on per-period gains T from renegotiating ownership introduced at the end of section 2. The next theorem characterizes *constrained relational efficient* ownership structures and the corresponding transfer payments if players can renegotiate ownership costlessly. We use the lable "constrained relational efficient" – the constraint being grim trigger strategies – because grim-trigger strategies generally are not optimal.

Theorem 3 *If parties support co-operation by means of grim-trigger strategies, then ownership structure ω is "constrained relational efficient" for the repeated interaction iff it minimizes aggregated betray payoffs, that is*

$$\omega \in \Omega^{CRE} = \left\{\omega \,\middle|\, B(\omega) \leq B(\omega') \;\; \forall \omega' \in \Omega \right\}.$$

The range of supporting discount factors is maximal by paying the transfer

$$\theta^{CRE} = \frac{(b_1(\omega) - c_1(\omega))\,(b_2(\omega) - d_2(\omega) - S) - (b_2(\omega) - c_2(\omega))\,(b_1(\omega) - d_1(\omega) - T)}{b_1(\omega) - d_1(\omega) + b_2(\omega) - d_2(\omega) - 2T}.$$

where $T = \frac{1}{2}(D^* - D(\omega))$.

The basic intuition behind this result is that the assumption of grim trigger strategies together with renegotiation to the short term efficient ownership structure removes the potential influence of ownership on the "punishment concern", i.e. payoffs in the punishment phase. Therefore, constrained efficient ownership optimizes only the ownership effects on the "deviation concern" which are short run gains from defections. In other words, an ownership structure is constrained relational efficient if its aggregated incentives to deviate $B(\omega)$ are minimal. Since ownership will change anyway if anything goes wrong the current ownership structure does not affect punishment payoffs.

The following little thought experiment shows that the aggregated deviation concerns $B(\omega)$ are related to the size of the hold-up problem if utility functions $u_i((e_i, e_j), \omega)$ are single peaked and continuous which is only a slighly stronger assumption than supposing a unique best response function as we did. Remember that a short term efficient ownership structure ω minimizes the size $HP(\omega) = \sum_{i=1,2} u_i(e^c, \omega) - u_i\left(e^d(\omega), \omega\right)$ of the hold-up problem. Now imagine a world with only two ownership structures ω_0, ω_1 to choose from, one characterized by a negligable holdup problem $HP(\omega_0) \to 0$ and another with a big one $HP(\omega_1) \gg 0$. In turn continuity and single peakedness of $u_i((e_i, e_j), \omega)$ imply that $u_i((e_i^d(\omega_0), e_j^c), \omega_0) \to u_i(e^c, \omega_0)$ and therefore a small hold-up generates small aggregated incentives $B(\omega_0) = \sum_{i=1,2} u_i((e_i^d(\omega_0), e_j^c), \omega_0) - u_i(e^c, \omega_0) \to 0$ to defect in ownership structure ω_0 with negligable holdup problem in contrast to ω_1. This example demonstrates that there are reasonable conditions under which short term efficient and constrained relational efficient ownership structures can coincide.[20] However, this reasoning shows at the same time that the underlying causes for this coincidence are quite different, so that with more than two ownership structures to choose from it gets more likely that these principles will not be in line with each other.

Sum up. The following little table sums up the results of this theory.

ownership concept	characterization	interpretation	proponents
short term efficient	minimize $HP(\omega)$	minimize holdup problem	HM
relational efficient no ren	minimze $\frac{B(\omega)-S^*}{B(\omega)-V(\omega)}$	optimal enforcement	we, Halonen
relational efficient with ren	minimize $\frac{B(\omega)-S^*}{B(\omega)-V(\omega)}$	optimal enforcement	we
constr rel eff with ren	minimize $B(\omega)$	minimize deviation concern	G, BGM

4 Property Rights

Our main object of interest in this article are ownership structures. So far we have not assumed any structure on them. Therefore, the basic structure of this theory holds for arbitrary exogenous parameters that influence bargaining positions (or the threat point if bargaining breaks down). This could be any other part of the institutional and legal framework or environmental and technological conditions etc. In this section, however, we want to be more specific about property rights, since in many real situations "ownership rights" can be decomposed into "asset ownership" as promoted by Hart and Moore (1990).

[20] This explains why for example Bragelien (2002) – who maintains Baker, Gibbons and Murphy's assumptions on a binary set of ownership structures, renegotiation and grim trigger strategies – concludes that there is a tendency for short term optimal ownership structures to be also optimal for relational contracts.

Let A denote a set of nonhuman assets (machines, buildings, land, client lists, patents, copy rights, etc.).

Definition 4 *A partition* $\omega = (A_1, A_2, A_{12})$ *is called* "two-party-ownership-structure"*. The subset* A_i *are privately owned assets of party* i *and* A_{12} *are jointly owned assets.*

Our interpretation of ownership follows the tradition of Hart and Moore. Ownership of an asset is defined as veto power over the use of the asset. Joint ownership means that every owner has veto power, i.e. a jointly owned asset can only be used by consent of all owners. In contrast to agents' actions that are observable but not verifiable ownership structure ω is assumed observable and verifiable in court. The following definition differentiates several cases.

Definition 5 *We will call a two party ownership structure* $\omega = (A_1, A_2, A_{12})$:

1. Joint Ownership (J), *if all assets are owned jointly* $\omega^J = (\emptyset, \emptyset, A)$ *or* $A_{12} = A$ *and* $A_i = \emptyset$ *for* $i = 1, 2$*;*
2. Integration (I), *if one party owns all assets* $\omega^I = (A, \emptyset, \emptyset)$ *or* $A_1 = A$ *and* $A_2 = A_{12} = \emptyset$*;*
3. Outsourcing (O), *if there are no jointly owned assets and both parties own assets* $\omega^O = (A_1, A_2, \emptyset)$ *or* $A_{12} = \emptyset, A_i \neq \emptyset$ *for* $i = 1, 2$*;*
4. Mixed Ownership (M), *if there are privately owned assets for at least one party, say 1, and jointly owned assets* $\omega^M = (A_1, A_2, A_{12})$*, and* $A_1, A_{12} \neq \emptyset$*.*

In a sense mixed ownership is the generic case since every other ownership structure can be approximated by a converging sequence of mixed ownership structures. The remainder of this article will be devoted to the question which of the previously defined ownership structures are relational efficient under different specifications of the model.

5 Specific Models

In this section we compare our results on relational efficient ownership with Garvey (1995) and Halonen (2002) both of which are specific versions of the following structure:

$$\begin{aligned}
E_i &= \mathbb{R}_+ \\
Q(e_1, e_2) &= q_1 e_1 + q_2 e_2 \\
P_i(e, \omega) &= p_i e_i + r_{-i} e_{-i} \text{ with } p_i + r_i \leq q_1 + q_2 \\
p_i, q_i, r_i &\in \mathbb{R}_+
\end{aligned}$$

and thereby

$$u_i(e,(\lambda_1,\lambda_2)) = \frac{1}{2}((q_i+\lambda_i)e_i + (q_{-i}-\lambda_{-i})e_{-i}) - C_i(e_i)$$

where $\lambda_i \equiv p_i - r_i \in [-q_i, q_i]$ for $i = 1,2$. The boundary case $(\lambda_1,\lambda_2) = (q_1,q_2)$ implies $p_i = q_i$ and $r_i = 0$. In this formulation this is the unique case where the hold-up problem disappears and there is no gain in forming a relationship or $Q(e_1,e_2) - P_1(e,\omega) - P_2(e,\omega) = 0$. Cost functions are power functions given as

$$C_i(e_i) = k_i e_i^\gamma$$

with $\gamma > 1$. Ownership structures Ω are parametrized as subsets of 2-vectors

$$\omega = (\lambda_1,\lambda_2) \in \Omega \subset [-q_1,q_1] \times [-q_2,q_2] \subset \mathbb{R}^2.$$

Note that in this formulation ownership structures at the same time determine the value of the relationship $Q(e_1,e_2) - P_1(e,\omega) - P_2(e,\omega)$ because they are directly defined by its consequences on disagreement payoffs $P_i(e,\omega)$. Clearly, it is not realistic to assume that all $(\lambda_1,\lambda_2) \in [-q_1,q_1] \times [-q_2,q_2]$ are available in practice. In order to design efficient ownership, however, it is an interesting exercise to see which constellation of disagreement payoffs induces relational efficient ownership.

Proposition 1 *Consider the set of ownership structures given by the 2-dimensional parameter space* $\Omega = [-q_1,q_1] \times [-q_2,q_2] \subset \mathbb{R}^2$.

1. *The relational efficient ownership structure* $(\lambda_1,\lambda_2)^{**}$ *generally differs from the short term efficient* $(\lambda_1,\lambda_2)^*$ *and depends on parameter values, in particular on the cost function parameter* γ. *Numerical computations suggest a pattern where*

$$\begin{aligned} (\lambda_1,\lambda_2)^{**} &= (\lambda_1,\lambda_2)^* \text{ for } \gamma < 2 \\ (\lambda_1,\lambda_2)^{**} &\neq (\lambda_1,\lambda_2)^* \text{ for } \gamma > 2. \end{aligned}$$

2. *If the cost function is a power function the short term efficient ownership structure and the* constrained *relational efficient ownership structure coincide. Formally,* $\forall \gamma > 0$, $(\lambda_1,\lambda_2)^* = (\lambda_1,\lambda_2)^{CRE}$.

3. *For quadratic cost functions ownership is irrelevant, that is all ownership structures* $(\lambda_1,\lambda_2) \in \Omega$ *including the whole set of mixed ownership structures are relational efficient. Formally,* $\gamma = 2 \Rightarrow \Omega^{**} = \Omega$.

For the first claim one has to identify cases where relational efficiency and short term efficiency do not identify the same ownership structure. We do this by numerical examples (see proof in the appendix). These computations show that the pattern based on the cost function parameter γ identified by Halonen (2002) is even more general. Claim 2 shows that in this simple framework removing the effects of ownership on punishments ensures that short term efficient structures are also constrained relational efficient. Finally, claim 3 tells us that quadratic cost functions have very special (non-generic) implications, which makes it unfortunate that Garvey and also Baker, Gibbons and Murphey focus much of their analysis on just this case.

The set of ownership structures we investigated here in proposition 1 is larger than the one in the original papers, where a finite (Halonen) or one-dimensional (Garvey) subset of $[-q_1, q_1] \times [-q_2, q_2]$ is considered. We think that Garvey's continuum set of ownership structures is less intuitive than Halonen's binary set. Nevertheless, Garvey's contribution was seminal by bringing up the subject at a time when the literature on property rights was dominated by incomplete contract theory and static models.

Garvey's Model. The subspecifications in Garvey's model are

$$\begin{aligned} Q(e_1, e_2) &= e_1 + e_2 \\ C_i(e_i) &= \frac{1}{2\alpha_i} e_i^2 \text{ with } \alpha_1 = \alpha, \alpha_2 = 1 - \alpha \\ P_i(e, \omega) &= \rho_i (e_1 + e_2) \text{ with } \rho_1 = \rho, \rho_2 = 1 - \rho \\ \rho &\in \Omega = [0, 1] \text{ continuum of ownership structures} \end{aligned}$$

that is $q_1 = q_2 = 1$, $p_1 = r_2 = \rho$, $p_2 = r_1 = 1 - \rho$, $\lambda_1 = -\lambda_2 = 2\rho - 1$ and for the cost function $k_1 = \frac{1}{2\alpha_i}, \gamma = 2$. Applying our theory (theorems 1, 2, and 3) reformulates and extends Garvey's results.[21]

Proposition 2 *(i) In Garvey's specification, independent of what can be renegotiated, ownership is irrelevant for relational efficiency:* $\Omega^{**} = \Omega$.

(ii) If instead in Garvey's specification agents are restricted to grim trigger strategies and only ownership can be renegotiated, then the short term efficient ownership structure and the constrained relational efficient ownership structure coincide:

$$\Omega^* = \Omega^{CRE} = \{\alpha\}.$$

[21] The reason why claim (ii) of the following proposition does not replicate Garvey's claims that relational contracts require more symmetric property rights is that Garvey assumes an exogenous and non-efficient transfer $\theta = \rho c_2 - (1 - \rho) c_1 \neq \theta^{CRE}$, while we allow agents to choose the transfer scheme optimally.

Halonen's Model. In contrast to Garvey, Halonen only compares joint ownership ω^J with full integration ω^I. Her specifications within this formulation are

$$\begin{aligned}
\omega &\in \Omega = \left\{\omega^J, \omega^I\right\} = \{0, \lambda\} \\
\omega^J &= 0 = \text{Joint Ownership} \\
\omega^I &= \lambda = \text{Integration with } \lambda \in [0,1] \\
Q(e_1, e_2) &= e_1 + e_2 \\
P_1(e, \omega) &= \omega e_1 \\
P_2(e, \omega) &= 0,
\end{aligned}$$

that is $q_1 = q_2 = 1$, $p_2 = r_1 = r_2 = 0$, $p_1 = \omega$ and for the cost structure $k_1 = 1, \gamma > 1$ or

$$C_i(e_i) = e_i^\gamma \text{ with } \gamma > 1$$

Lemma 1 *The short term efficient ownership structure defined by (3d) is given by full integration:* $\omega^* = \omega^I$.

Again, applying our theory to Halonen's specification supports and generalizes Halonen's observation that joint ownership may be optimal in a dynamic investment relation.

Proposition 3 *(i) Suppose agents can choose optimally equilibrium strategies. Then independent of what can be renegotiated, in Halonen's specification the relational efficient ownership structure is*

$$\Omega^{**} = \begin{cases} \omega^I & \textit{for} \quad \gamma \in (1,2) \\ \left\{\omega^J, \omega^I\right\} & \textit{for} \quad \gamma = 2 \\ \omega^J & \textit{for} \quad \gamma > 2 \end{cases},$$

where $C_i(e_i) = e_i^\gamma$ *with* $\gamma > 1$.

(ii) If instead agents are restricted to use grim trigger strategies and ownership can be renegotiated, then the constrained relational efficient ownership structure coincides with the short term efficient ownership structure, and is full integration: $\Omega^{CRE} = \left\{\omega^I\right\}$.

6 Conclusions

We developed a model for the analysis of the efficient allocation of property rights in long term relations, where investment levels are not contractible and must be sustained in equilibrium. Applying insights from repeated game analysis to this framework not only generalizes the previous literature on the subject but at the same time simplifies the results. First, we generalize previous models in various dimensions: (i) we do not single out specific ownership structures to be compared, (ii) we do not assume specific functional forms for production or cost functions,

(iii) we allow for both possibilities with and without renegotiation and, (iv) we do not impose assumptions on the strategies of business parties' non-contractible investment decisions. Second, in our more general framework one would expect a richer and more complicated pattern of results. However, our results are surprisingly simple and straightforward. We reinforce the previous literature on the subject in the observation that an ownership structure that is short term efficient is not neccessarily long term efficient. In relevant situations those criteria do not coincide. In particular, where short term efficiency points to ownership structures that minimize the holdup problem relational efficient ownership structures are designed to render a relationship as stable as possible in the sense that incentives to misbehave are minimized by the threat of the most severe punishment that is available in a non-contractible environment. Second, our results are simple compared to previous models as this optimal off-equilibrium punishment threat is not compromized by the possibility to renegotiate since rational business partners recognize that it is always more efficient to continue the relationship and restitute eventual foregone misbehavior by appropriate transfer payments.

Our results confirm that in a dynamic world the optimal allocation of property rights does not in general coincide with the static one identified by the Hart-Moore paradigm, but we identify relevant situations where it does.

7 Appendix A: Proofs

Theorem 1. **Proof.** Incentive compatibility is given by

$$\begin{aligned}\frac{1}{1-\delta}\left(c_i(\omega)+\theta_i\right) &\geq b_i(\omega)+\frac{\delta}{1-\delta}v_i(\omega) \Leftrightarrow \\ c_i(\omega)+\theta_i &\geq (1-\delta)\,b_i(\omega)+\delta v_i(\omega)\end{aligned}$$

or

$$\underline{\delta}_i = \frac{b_i(\omega)-c_i(\omega)-\theta_i}{b_i(\omega)-v_i(\omega)}.$$

Incentive compatibility is satisfied for both players if no player has a discount factor below

$$\underline{\delta} = \max\left\{\underline{\delta}_1, \underline{\delta}_2\right\}.$$

The "optimal transfer" θ^{**} minimizes $\underline{\delta}$ and hence satisfies

$$\frac{b_1(\omega)-c_1(\omega)-\theta^{**}}{b_1(\omega)-v_1(\omega)} = \frac{b_2(\omega)-c_2(\omega)+\theta^{**}}{b_2(\omega)-v_2(\omega)}$$

this yields

$$\theta^{**}(\omega) = \frac{\left(b_1(\omega)-c_1(\omega)\right)\left(b_2(\omega)-v_2(\omega)\right)-\left(b_2(\omega)-c_2(\omega)\right)\left(b_1(\omega)-v_1(\omega)\right)}{b_2(\omega)-v_2(\omega)+b_1(\omega)-v_1(\omega)} \tag{6}$$

and

$$\underline{\delta}^{**}(\omega) = \frac{b_1(\omega)-c_1(\omega)+b_2(\omega)-c_2(\omega)}{b_1(\omega)-v_1(\omega)+b_2(\omega)-v_2(\omega)} \tag{7}$$

or

$$\underline{\delta}^{**}(\omega) = \frac{B(\omega)-S^*}{B(\omega)-V(\omega)}. \tag{8}$$

■

Theorem 2. **Proof.** We show (i) that there exists a subgame perfect equilibrium supporting indefinite co-operation on its equilibrium path if and only if $\delta \geq \underline{\delta}^{**}(\omega) = \frac{B(\omega)-S^*}{B(\omega)-V(\omega)}$ and $\theta = \theta^{**}(\omega)$ from theorem 1 and (ii) that all continuation payoffs are Pareto efficient which leaves no room for renegotiation. The proof proceeds by defining 'restitution' strategies as in Levin (1998, 2003), and then to show that these strategies satisfy the required properties.

Definition 6 *Strategies:*[22]

Start playing from Phase 1.

[22] The strategies are inspired by – but slightly different from (due to monetary transfers) – by those discussed in van Damme (1989), Farrell and Maskin (1989) and Segerström (1988). "Asymmetry" refers to the different behavior of the "defector" and the "afflicted party" off equilibrium, in contrast to grim trigger strategies where both parties (also the defector) punish. Asymmetry does not mean that players in the same role (e.g. defector) behave differently.

Phase 1:

Invest efficiently e_i^c *and pay the equilibrium transfer* θ*; if an agent* i *deviates, start Phase 2.*

Phase 2:

Agent $j \neq i$ *: If at the beginning of the period you receive transfer* F^{ij} *from agent* i*, go back to Phase 1; otherwise choose* $e_j = 0$ *and start again Phase 2.*

Agent i *: Pay fine* F^{ij} *to agent* j *and go back to Phase 1.*

If a player deviates in Phase 2, re-starts Phase 2 against that player.

We first show that these strategies with maximal F^{ij} are a SPNE and constitute an optimal punishment. First note that $v_i(\omega) = P_i\left(e_i^b(0), e_j = 0, \omega\right)$ and that defecting during Phase 2 does not increase agent i's continuation payoff. Now note that for player j, defecting in Phase 2 is not profitable if

$$(1-\delta) P_j\left(e_i^b(0), e_j = 0, \omega\right) + \delta S^* \geq (1-\delta) P_j(e_i^b(0), e_j^b(e_i^b(0)), \omega),$$

which is always satisfied when $\delta > \underline{\delta}^{**}$, which implies that if the strategies constitute an equilibrium, the equilibrium is subgame perfect.

Now note that for any given equilibrium transfer θ, if agent i defects unilaterally from restitution strategies with $F^{ij} = \frac{c_i - v_i(\omega) + \theta}{1-\delta}$, he expects $b_i(\omega)$ from the period of the defection and $\delta OP_i(\omega) = \frac{\delta}{1-\delta} v_i(\omega)$ from the rest of the game. Hence, a defection is deterred when $c_i(\omega) + \theta \geq (1-\delta) b_i(\omega) + \delta v_i(\omega)$, which is exactly the condition relevant with optimal punishments and no renegotiation of ownership, hence these strategies are optimal and induce the same optimal transfer payment θ^{**}.

We now show that the equilibria in restitution strategies with maximal F^{ij} are Pareto efficient. Since the strategies specify that a deviator i always pays the fine required to reestablish co-operation and the afflicted party j alwas returns to co-operation all continuation payoffs are efficient in the sense that by definition of co-operation joint payoffs cannot be increased. Suppose $\omega \in \Omega^{**}$ (as defined in theorem 1). After a defection by i, before the punishment phase starts i can propose j to modify ownership structure to $\omega' \neq \omega$. However, in the subgame after agent i defects from the given equilibrium, agent j's subgame equilibrium payoff at the beginning of phase 2 is the entire remaining value of the relationship (net surplus from the relation with efficient investment). Since $\omega \in \Omega^{**}$, this is strictly greater than any payoff player j could obtain by renegotiating ownership using Nash bargaining, hence – even though $z = 0$ – player j has no incentive to agree to any modification to ω' and therefore renegotiation of ownership will not occur. ■

Theorem 3. **Proof.** Incentive compatibility for grim trigger strategies

$$c_i(\omega) + \theta_i \geq (1-\delta) b_i(\omega) + \delta (d_i(\omega) + T)$$

yields

$$\underline{\delta}_i = \frac{b_i(\omega) - c_i(e^c, \omega) - \theta_i}{b_i(\omega) - d_i(\omega) - T}$$

and

$$\underline{\delta} = \max\{\underline{\delta}_1, \underline{\delta}_2\}$$

The "optimal sharing rule" θ^{CRE} minimizes $\underline{\delta}$ and hence satisfies

$$\frac{b_1(\omega) - c_1(\omega) - \theta^{CRE}}{b_1(\omega) - d_1(\omega) - T} = \frac{b_2(\omega) - c_2(\omega) + \theta^{CRE}}{b_2(\omega) - d_2(\omega) - T}$$

this yields

$$\theta^{CRE} = \frac{(b_1(\omega) - c_1(\omega))(b_2(\omega) - d_2(\omega) - T) - (b_2(\omega) - c_2(\omega))(b_1(\omega) - d_1(\omega) - T)}{b_1(\omega) - d_1(\omega) + b_2(\omega) - d_2(\omega) - 2T}$$

and

$$\begin{aligned}
\underline{\delta}^{CRE} &= \frac{b_1(\omega) - c_1(\omega) - \theta^{CRE}}{b_1(\omega) - d_1(\omega) - T} \\
&= \frac{b_1(\omega) - c_1(\omega) - \frac{(b_1(\omega)-c_1(\omega))(b_2(\omega)-d_2(\omega)-T)-(b_2(\omega)-c_2(\omega))(b_1(\omega)-d_1(\omega)-T)}{b_1(\omega)-d_1(\omega)+b_2(\omega)-d_2(\omega)-2T}}{b_1(\omega) - d_1(\omega) - T} \\
&= \frac{b_1(\omega) - c_1(\omega) + b_2(\omega) - c_2(\omega)}{b_1(\omega) - d_1(\omega) + b_2(\omega) - d_2(\omega) - 2T} \\
\underline{\delta}^{CRE} &= \frac{b_1(\omega) - c_1(\omega) + b_2(\omega) - c_2(\omega)}{b_1(\omega) - d_1^* + b_2(\omega) - d_2^*}
\end{aligned}$$

or

$$\underline{\delta}^{CRE}(\omega) = \frac{B(\omega) - S^*}{B(\omega) - D^*}. \tag{9}$$

$\underline{\delta}^{CRE}(\omega)$ strictly increases with $B(\omega)$ since $S^* > D^*$. This proves theorem 3. ■

Proposition 1. **Proof.** This specification yields $e_i^c = \left(\frac{q_i}{\gamma k_i}\right)^{\frac{1}{\gamma-1}}$, $e_i^d = \left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}}$ and

$$\begin{aligned}
c_i(\lambda_1, \lambda_2) &= \frac{1}{2}\left((q_i + \lambda_i) e_i^c + (q_{-i} - \lambda_{-i}) e_{-i}^c\right) - k_i (e_i^c)^\gamma \\
&= \frac{1}{2}\left((q_i + \lambda_i)\left(\frac{q_i}{\gamma k_i}\right)^{\frac{1}{\gamma-1}} + (q_{-i} - \lambda_{-i})\left(\frac{q_{-i}}{\gamma k_{-i}}\right)^{\frac{1}{\gamma-1}}\right) - k_i \left(\frac{q_i}{\gamma k_i}\right)^{\frac{\gamma}{\gamma-1}}
\end{aligned}$$

and

$$\begin{aligned}
S^* &= q_1\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}} + q_2\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}} - k_1\left(\frac{q_1}{\gamma k_1}\right)^{\frac{\gamma}{\gamma-1}} - k_2\left(\frac{q_2}{\gamma k_2}\right)^{\frac{\gamma}{\gamma-1}} \\
&= q_1\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}} + q_2\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}} - \frac{q_1}{\gamma}\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}} - \frac{q_2}{\gamma}\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}} \\
&= \frac{(\gamma-1)}{\gamma}\left[q_1\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}} + q_2\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}}\right] \\
d_i(\lambda_1,\lambda_2) &= \frac{1}{2}\left((q_i+\lambda_i)\,e_i^d + (q_{-i}-\lambda_{-i})\,e_{-i}^d\right) - k_i\left(e_i^d\right)^{\gamma} \\
&= \frac{1}{2}\left((q_i+\lambda_i)\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}} + (q_{-i}-\lambda_{-i})\left(\frac{q_{-i}+\lambda_{-i}}{2\gamma k_{-i}}\right)^{\frac{1}{\gamma-1}}\right) - k_i\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{\gamma}{\gamma-1}}
\end{aligned}$$

Further, we obtain

$$\begin{aligned}
D(\lambda_1,\lambda_2) &= V(\lambda_1,\lambda_2) = d_1(\omega) + d_2(\omega) = \\
&= q_1\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{1}{\gamma-1}} - k_1\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{\gamma}{\gamma-1}} + q_2\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{1}{\gamma-1}} - k_2\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{\gamma}{\gamma-1}} \\
&= \left(q_1 - \frac{q_1+\lambda_1}{2\gamma}\right)\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{1}{\gamma-1}} + \left(q_2 - \frac{q_2+\lambda_2}{2\gamma}\right)\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{1}{\gamma-1}}
\end{aligned}$$

For the short term efficient ownership structure first order conditions yield

$$\begin{aligned}
\frac{\partial D(\lambda_1,\lambda_2)}{\partial \lambda_i} &= \frac{\partial}{\partial \lambda_i}\left(q_1\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{1}{\gamma-1}} - k_1\left(\frac{q_i+\lambda_1}{2\gamma k_1}\right)^{\frac{\gamma}{\gamma-1}}\right) \\
&= \frac{q_i-\lambda_i}{2(\gamma-1)(q_i+\lambda_i)}\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}} = 0 \text{ for} \\
\lambda_i &= q_i.
\end{aligned}$$

Next, calculate

$$\begin{aligned}
b_i(\lambda_1,\lambda_2) &= \frac{1}{2}\left((q_i+\lambda_i)\,e_i^d + (q_{-i}-\lambda_{-i})\,e_{-i}^c\right) - k_i\left(e_i^d\right)^{\gamma} \\
&= \frac{1}{2}\left((q_i+\lambda_i)\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}} + (q_{-i}-\lambda_{-i})\left(\frac{q_{-i}}{\gamma k_{-i}}\right)^{\frac{1}{\gamma-1}}\right) - k_i\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{\gamma}{\gamma-1}} \\
&= (\gamma-1)\,k_i\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{\gamma}{\gamma-1}} + \frac{1}{2}(q_{-i}-\lambda_{-i})\left(\frac{q_{-i}}{\gamma k_{-i}}\right)^{\frac{1}{\gamma-1}}
\end{aligned}$$

and

$$\begin{aligned}
B(\lambda_1,\lambda_2) &= b_1(\omega)+b_2(\omega)\\
&= (\gamma-1)\,k_1\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{\gamma}{\gamma-1}}+\frac{1}{2}(q_2-\lambda_2)\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}}\\
&\quad+(\gamma-1)\,k_2\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{\gamma}{\gamma-1}}+\frac{1}{2}(q_1-\lambda_1)\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}}\\
&= (\gamma-1)\,\frac{q_1+\lambda_1}{2\gamma}\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{1}{\gamma-1}}+\frac{1}{2}(q_1-\lambda_1)\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}}\\
&\quad+(\gamma-1)\,\frac{q_2+\lambda_2}{2\gamma}\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{1}{\gamma-1}}+\frac{1}{2}(q_2-\lambda_2)\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}}
\end{aligned}$$

To calculate the constrained relational efficient ownership structure apply theorem 3 and calculate first order conditions:

$$\begin{aligned}
\frac{\partial B(\lambda_1,\lambda_2)}{\partial\lambda_i} &= \frac{\partial}{\partial\lambda_i}\left((\gamma-1)\,\frac{q_i+\lambda_i}{2\gamma}\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}}+\frac{1}{2}(q_i-\lambda_i)\left(\frac{q_i}{\gamma k_i}\right)^{\frac{1}{\gamma-1}}\right)\\
&= \frac{1}{2}\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}}-\frac{1}{2}\left(\frac{q_i}{\gamma k_i}\right)^{\frac{1}{\gamma-1}}\\
&= \frac{1}{2}\left(\left(\frac{q_i+\lambda_i}{2\gamma k_i}\right)^{\frac{1}{\gamma-1}}-\left(\frac{q_i}{\gamma k_i}\right)^{\frac{1}{\gamma-1}}\right)=0 \text{ for}\\
\frac{q_i+\lambda_i}{2\gamma k_i} &= \frac{q_i}{\gamma k_i}\Leftrightarrow\lambda_i=q_i.
\end{aligned}$$

This is identical with the short term efficient ownership structure and hence shows claim 2 of proposition 1. To analyze relational efficiency theorems 1 and 2 yield

$$\underline{\delta}^{**}(\lambda_1,\lambda_2)=\frac{B(\lambda_1,\lambda_2)-S^*}{B(\lambda_1,\lambda_2)-V(\lambda_1,\lambda_2)}=$$

$$=\frac{\frac{(\gamma-1)(q_1+\lambda_1)}{\gamma}\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{1}{\gamma-1}}+\frac{2q_1-\gamma(q_1+\lambda_1)}{\gamma}\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}}+\frac{(\gamma-1)(q_2+\lambda_2)}{\gamma}\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{1}{\gamma-1}}+\frac{2q_2-\gamma(q_2+\lambda_2)}{\gamma}\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}}}{(q_1-\lambda_1)\left(\left(\frac{q_1}{\gamma k_1}\right)^{\frac{1}{\gamma-1}}-\left(\frac{q_1+\lambda_1}{2\gamma k_1}\right)^{\frac{1}{\gamma-1}}\right)+(q_2-\lambda_2)\left(\left(\frac{q_2}{\gamma k_2}\right)^{\frac{1}{\gamma-1}}-\left(\frac{q_2+\lambda_2}{2\gamma k_2}\right)^{\frac{1}{\gamma-1}}\right)}$$

To show claim 1. note that maximization of $D(\lambda_1,\lambda_2)$ and minimization of $\frac{B(\lambda_1,\lambda_2)-S^*}{B(\lambda_1,\lambda_2)-V(\lambda_1,\lambda_2)}$ may yield different results. The following pictures show $\underline{\delta}^{**}(\lambda_1,\lambda_2)=\frac{B(\lambda_1,\lambda_2)-S^*}{B(\lambda_1,\lambda_2)-V(\lambda_1,\lambda_2)}$ for two salient cost function specifications and $q_1=q_2=1$.

Finally, for claim 3 the special case $\gamma=2$ expressions simplify to

$$
\begin{aligned}
e_i^c &= \frac{q_i}{2k_i} \\
e_i^d &= \frac{q_i+\lambda_i}{4k_i} \\
c_i(\lambda_1,\lambda_2) &= \frac{1}{2}\left((q_i+\lambda_i)\frac{q_i}{2k_i}+(q_{-i}-\lambda_{-i})\frac{q_{-i}}{2k_{-i}}\right)-k_i\left(\frac{q_i}{2k_i}\right)^2 \\
S^* &= \frac{q_1^2}{4k_1}+\frac{q_2^2}{4k_2} \\
d_i(\lambda_1,\lambda_2) &= \frac{1}{2}\left((q_i+\lambda_i)\frac{q_i+\lambda_i}{4k_i}+(q_{-i}-\lambda_{-i})\frac{q_{-i}+\lambda_{-i}}{4k_{-i}}\right)-k_i\left(\frac{q_i+\lambda_i}{4k_i}\right)^2 \\
&= \frac{(q_i+\lambda_i)^2}{16k_i}+\frac{q_{-i}^2-\lambda_{-i}^2}{8k_{-i}}
\end{aligned}
$$

and thereby

$$
\begin{aligned}
D(\lambda_1,\lambda_2) &= q_1\frac{q_1+\lambda_1}{4k_1}+q_2\frac{q_2+\lambda_2}{4k_2}-k_1\left(\frac{q_1+\lambda_1}{4k_1}\right)^2-k_2\left(\frac{q_2+\lambda_2}{4k_2}\right)^2 \\
&= \frac{(q_1+\lambda_1)^2}{16k_1}+\frac{q_2^2-\lambda_2^2}{8k_2}+\frac{(q_2+\lambda_2)^2}{16k_2}+\frac{q_1^2-\lambda_1^2}{8k_1} \\
&= \frac{3q_1^2+2q_1\lambda_1-\lambda_1^2}{16k_1}+\frac{3q_2^2+2q_2\lambda_2-\lambda_2^2}{16k_2}
\end{aligned}
$$

and

$$
\begin{aligned}
b_i(\lambda_1,\lambda_2) &= \frac{(q_i+\lambda_i)^2}{8k_i}+(q_{-i}-\lambda_{-i})\frac{q_{-i}}{4k_{-i}}-k_i\left(\frac{q_i+\lambda_i}{4k_i}\right)^2 \\
&= \frac{(q_i+\lambda_i)^2}{16k_i}+(q_{-i}-\lambda_{-i})\frac{q_{-i}}{4k_{-i}} \\
B(\lambda_1,\lambda_2) &= \frac{(q_1+\lambda_1)^2}{16k_1}+(q_2-\lambda_2)\frac{q_2}{4k_2}+\frac{(q_2+\lambda_2)^2}{16k_2}+(q_1-\lambda_1)\frac{q_1}{4k_1} \\
&= \frac{(q_1+\lambda_1)^2+4q_1(q_1-\lambda_1)}{16k_1}+\frac{(q_2+\lambda_2)^2+4q_2(q_2-\lambda_2)}{16k_2} \\
&= \frac{5q_1^2-2q_1\lambda_1+\lambda_1^2}{16k_1}+\frac{5q_2^2-2q_2\lambda_2+\lambda_2^2}{16k_2}.
\end{aligned}
$$

Now plug in and obtain

$$
\begin{aligned}
\frac{B(\lambda_1,\lambda_2)-S^*}{B(\lambda_1,\lambda_2)-V(\lambda_1,\lambda_2)} &= \frac{\frac{(q_1+\lambda_1)^2}{8k_1}+\frac{2q_1-2(q_1+\lambda_1)}{2}\frac{q_1}{2k_1}+\frac{(q_2+\lambda_2)^2}{8k_2}+\frac{2q_2-2(q_2+\lambda_2)}{2}\frac{q_2}{2k_2}}{(q_1-\lambda_1)\left(\frac{q_1}{2k_1}-\frac{q_1+\lambda_1}{4k_1}\right)+(q_2-\lambda_2)\left(\frac{q_2}{2k_2}-\frac{q_2+\lambda_2}{4k_2}\right)} \\
&= \frac{\frac{(q_1+\lambda_1)^2-4q_1\lambda_1}{8k_1}+\frac{(q_2+\lambda_2)^2-4q_2\lambda_2}{8k_2}}{\frac{2(q_1-\lambda_1)^2}{8k_1}+\frac{2(q_2-\lambda_2)^2}{8k_2}} \\
&= \frac{1}{2}
\end{aligned}
$$

which does not depend on (λ_1,λ_2). This proves the last claim of the proposition. ■

Proposition 2. Proof. (i) The utility function (1) becomes

$$u_i(e,\rho) = \rho_i(e_1+e_2) - \frac{1}{2\alpha_i}e_i^2$$

This yields $e_i^c = \alpha_i, e_i^d = \rho_i\alpha_i$ and for Garvey's specification the crucial parameters (3a to 3g) of the stage game are given by

$$b_i = \frac{1}{2}\rho_i^2\alpha_i + \rho_i(1-\alpha_i) \tag{10a}$$

$$c_i = \rho_i - \frac{1}{2}\alpha_i \tag{10b}$$

$$d_i = \frac{1}{2}\rho_i^2\alpha_i + \rho_i(1-\rho_i)(1-\alpha_i) \tag{10c}$$

$$S^* = \frac{1}{2} \tag{10d}$$

$$B(\rho) = \frac{1}{2}\left(1+\alpha+\rho^2\right) - \rho\alpha \tag{10e}$$

$$D(\rho) = V(\rho) = \frac{1}{2}\left(1-\alpha-\rho^2\right) + \rho\alpha \tag{10f}$$

As we have seen in the proof of proposition 1 plugging in parameters (10a - 10f) into equation (8) we obtain

$$\begin{aligned}\underline{\delta}^{**}(\rho) &= \frac{B(\rho)-S^*}{B(\rho)-D(\rho)} \\ &= \frac{1}{2}\end{aligned}$$

which implies the first claim, $\Omega^N = \Omega$. Apply definition (3d) and theorem 3 to parameters (10a - 10f), and the second claim (ii) obtains. ■

Lemma 1. Proof. Halonen's specification yields

$$D(\omega) = \left(\frac{\gamma-1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}} + \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right) + \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}} + \frac{1-\omega}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}\right)\gamma^{\frac{1}{\gamma-1}}.$$

The lemma follows from $D(\omega)$ being increasing with ω:

$$\begin{aligned}D'(\omega) &= \frac{d}{d\omega}\left(\frac{\gamma-1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}} + \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right) + \left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}} + \frac{1-\omega}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}\right)\gamma^{\frac{1}{\gamma-1}} \\ &= \gamma^{\frac{1}{\gamma-1}}\left(\frac{1}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}} - \frac{1}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}} + \frac{1}{2}\frac{1-\omega}{2}\frac{1}{\gamma-1}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}-1}\right) \\ &= \gamma^{\frac{1}{\gamma-1}}\left(\frac{1}{2}\frac{1-\omega}{2}\frac{1}{\gamma-1}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}-1}\right) \\ &> 0 \text{ for } \omega < 1\end{aligned}$$

■

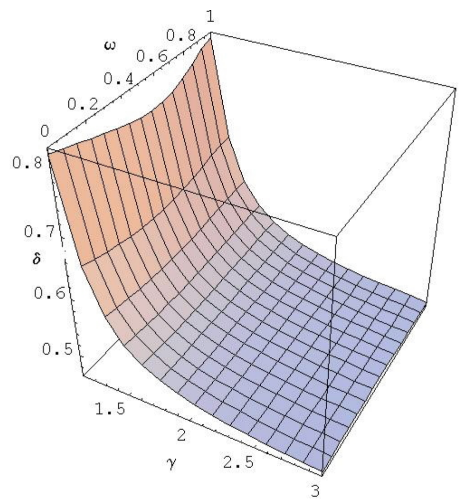

Figure 1: $\underline{\delta}^{**}(\omega,\gamma)$

Proposition 3. **Proof.** For Halonen's parameters equation (8) becomes

$$\begin{aligned}\underline{\delta}^{**}(\omega) &= \frac{B(\omega)-S^*}{B(\omega)-D(\omega)} \\ &= \frac{\frac{\gamma-1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}}+\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right)+\frac{2-\omega}{2}-\frac{2(\gamma-1)}{\gamma}}{\frac{2-\omega}{2}-\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}-\frac{1-\omega}{2}\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}}\end{aligned}$$

Figure 2 (and the corresponding projections for $\gamma<2,\gamma=2,\gamma>2$) show $\underline{\delta}^{**}(\omega,\gamma)$ and confirm the first result. For the second statement, apply theorem 3 and verify that $B(\omega)$ decreases with ω:

$$\begin{aligned}B'(\omega) &= \frac{d}{d\omega}\left(\frac{\gamma-1}{\gamma}\left(\left(\frac{1+\omega}{2}\right)^{\frac{\gamma}{\gamma-1}}+\left(\frac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right)+\frac{2-\omega}{2}\right)\gamma^{\frac{1}{\gamma-1}} \\ &= \frac{1}{2}\gamma^{\frac{1}{\gamma-1}}\left(\left(\frac{1+\omega}{2}\right)^{\frac{1}{\gamma-1}}-1\right) \\ &< 0 \text{ for } \omega<1.\end{aligned}$$

■

8 Appendix B: Baker, Gibbons and Murphy's Model

Baker, Gibbons and Murphy's model is not an example of our theory since it is richer than other models, including multi-tasking moral hazard (i.e. player's actions are not directly observable, only their stochastic consequences are). Therefore, parts of our theory do not apply without adjustment. However, the restriction to grim trigger strategies can be easily relaxed in Baker, Gibbons and Murphy's model. Here we shortly investigate how Baker, Gibbons and Murphy's (2001) and (2002) results would change by relaxing this restriction, and show that our earlier conclusions are not affected by the presence of moral hazard.

Baker, Gibbons and Murphy only compare outsourcing ω^O with full integration ω^I (there named "employment"). To be able to compare Baker, Gibbons and Murphy's formulation to our previous setup we add joint ownership ω^{JO}, that is $\Omega = \left\{\omega^O, \omega^I, \omega^{JO}\right\}$. In Baker, Gibbons and Murphy's specification only one party (interpreted as upstream party) can invest in the joint project. We can now confirm and extend the main proposition in Baker, Gibbons and Murphy (2002).

Proposition 4 *In Baker, Gibbons and Murphy's model when agents are free to choose strategies optimally (Assumption 2) asset ownership affects the parties' temptations to renege on a relational contract and at the same time the maximal punishment they may be subject to, and hence it affects whether a given relational contract is feasible. Moreover, joint ownership* ω^{JO} *dominates (is never less relational efficient than) outsourcing* ω^O *although it may be less efficient than integration, depending on further specification.*

The following proof contains a sketched reformulation of Baker, Gibbons and Murphy's model in our notation.

Proof. Baker, Gibbons and Murphy's specifications in our notation are

$$\begin{aligned}
e_1 &= \left\{ \begin{array}{c} \mathbf{a} = (a_1, ..., a_n) \in \mathbb{R}^n_+, \text{ a vector} \\ \text{interpreted as"multi-task actions"} \end{array} \right\} \text{upstream party} \\
e_2 &= \emptyset \text{ downstream party has no action in the investment stage} \\
C_1(\mathbf{a}) &= \text{upstream party cost function} \\
C_2 &= 0 \\
Q(a) &= \left\{ \begin{array}{ccc} Q_L + \Delta Q & \text{with probability} & q(\mathbf{a}) \\ Q_L & \text{with probability} & 1 - q(\mathbf{a}) \end{array} \right. \\
P_1(a, \omega) &= \left\{ \begin{array}{ccccc} P_L + \Delta P & \text{with probability} & p(\mathbf{a}) & \text{for} & \omega = \omega^O \\ P_L & \text{with probability} & 1 - p(\mathbf{a}) & \text{for} & \omega = \omega^O \\ 0 & \text{for} & \omega = \omega^I & & \end{array} \right.
\end{aligned}$$

with $\Delta Q = Q_H - Q_L > 0$ and $\Delta P = P_H - P_L > 0$. Although the upstream partie's actions are not observable the downstream party can observe the realizations of P_1 and Q. Hence, the transfer payment may depend on them. In this setup the transfer or the so called *relational compensation contract* is

$$\theta_1 = -\theta_2 = s + b_{jk}$$

where s denotes a fixed salary and b_{jk} is a bonus payment that depends on the realizations of $Q = Q_j$ and $P = P_k$ with $j, k = H, L$. Corresponding to our notation let $S^* = Q_L + q(e^c)\Delta Q - C_1(e^c)$ be the *expected* first best surplus that can be achieved by the first best action e^c. As without moral hazard also in this setup the relational efficient ownership structure crucially depends on optimal punishment continuation payoffs denoted as before by $U_i(\omega) = \frac{v_i(\omega)}{1-\delta}$. These differ[23] from those considered by Baker, Gibbons and Murphy and are given by

$$(v_1(\omega), v_2(\omega)) = \begin{cases} (0, Q_L) & \text{for} \quad \omega^I \\ \left(\hat{P}, 0\right) & \text{for} \quad \omega^O \\ (0,0) & \text{for} \quad \omega^{JO} \end{cases}$$

where $\hat{P} := \arg\max_{\mathbf{a}} P_L + p(\mathbf{a})\Delta P - C_1(\mathbf{a})$ is what the upstream party can guarantee himself (minmax payoff) if he owns the asset. Integration means that the downstream party owns the asset. Since the downstream party cannot invest the maximum she can guarantee herself is Q_L forever. That is, for ω^O, ω^I the asset can be used by the party that owns it. In contrast, under joint ownership ω^{JO} the optimal punishment continuation payoffs do not reflect any further use of the asset since the asset can only be used by consent. Optimal punishment alters the non-deviation constraints in Baker, Gibbons and Murphy which become

$$\begin{array}{cc} \begin{array}{c} b_{jk} + \frac{\delta}{1-\delta}\left(c_1\left(\omega^I\right) + b_{jk}\right) \geq 0 \\ -b_{jk} + \frac{\delta}{1-\delta}\left(c_2\left(\omega^I\right) - b_{jk}\right) \geq \frac{\delta}{1-\delta} Q_L \end{array} & \text{for} \quad \omega^I \\ \begin{array}{c} b_{jk} + \frac{\delta}{1-\delta}\left(c_1\left(\omega^O\right) + b_{jk}\right) \geq \frac{\delta}{1-\delta}\hat{P} \\ Q_j - b_{jk} + \frac{\delta}{1-\delta}\left(c_2\left(\omega^O\right) - b_{jk}\right) \geq 0 \end{array} & \text{for} \quad \omega^O \\ \begin{array}{c} b_{jk} + \frac{\delta}{1-\delta}\left(c_1\left(\omega^{JO}\right) + b_{jk}\right) \geq 0 \\ Q_j - b_{jk} + \frac{\delta}{1-\delta}\left(c_2\left(\omega^{JO}\right) - b_{jk}\right) \geq 0 \end{array} & \text{for} \quad \omega^{JO}. \end{array}$$

If these inequalities hold for all j and k they must hold for the largest and smallest values of b_{jk}. This yields

$$\begin{array}{cc} \begin{array}{c} -\min_{j,k=H,L} b_{jk} \leq \frac{\delta}{1-\delta}\left(c_1\left(\omega^I\right) + b_{jk}\right) \\ \max_{j,k=H,L} b_{jk} \leq \frac{\delta}{1-\delta}\left(c_2\left(\omega^I\right) - b_{jk}\right) - \frac{\delta}{1-\delta} Q_L \end{array} & \text{for} \quad \omega^I \\ \begin{array}{c} -\min_{j,k=H,L} b_{jk} \leq \frac{\delta}{1-\delta}\left(c_1\left(\omega^O\right) + b_{jk}\right) - \frac{\delta}{1-\delta}\hat{P} \\ \max_{j,k=H,L}\left(b_{jk} - Q_j\right) \leq \frac{\delta}{1-\delta}\left(c_2\left(\omega^O\right) - b_{jk}\right) \end{array} & \text{for} \quad \omega^O \\ \begin{array}{c} -\min_{j,k=H,L} b_{jk} \leq \frac{\delta}{1-\delta}\left(c_1\left(\omega^I\right) + b_{jk}\right) \\ \max_{j,k=H,L}\left(b_{jk} - Q_j\right) \leq \frac{\delta}{1-\delta}\left(c_2\left(\omega^O\right) - b_{jk}\right) \end{array} & \text{for} \quad \omega^{JO}. \end{array}$$

[23] Here Baker, Gibbons and Murphy's specific assumptions on limiting the set of strategies and renegotiation possibilities come into play.

Summing up over both parties and using $S^* = c_1(\omega)+c_2(\omega)$ the crucial necessary and sufficient conditions[24] for a self enforcing relational contract become

$$\begin{array}{rll} \max_{j,k=H,L} b_{jk} - \min_{j,k=H,L} b_{jk} \leq \frac{\delta}{1-\delta}\left(S^* - Q_L\right) & \text{for} & \omega^I \\ \max_{j,k=H,L}\left(b_{jk} - Q_j\right) - \min_{j,k=H,L} b_{jk} \leq \frac{\delta}{1-\delta}\left(S^* - \hat{P}\right) & \text{for} & \omega^O \\ \max_{j,k=H,L}\left(b_{jk} - Q_j\right) - \min_{j,k=H,L} b_{jk} \leq \frac{\delta S^*}{1-\delta} & \text{for} & \omega^{JO}. \end{array}$$

This shows that the left side (the "maximum total reneging temptation" *and* the right side (the punishment concern) depend on the ownership structure. In particular, since $\hat{P} \geq 0$ outsourcing ω^O is weakly dominated with respect to relational efficiency by joint ownership. ■

On the other hand, Baker, Gibbons and Murphy's (2001) result on the "impossibility for firms to mimic spot markets" turns out to depend on agents chosing grim trigger strategies, and not to apply in a more general formulation where parties can choose strategies optimally.

Proposition 5 *"Bringing the market inside the firm". Consider integration ownership structure $\omega = \omega^I$ interpreted as a firm. The relational contract that replicates the payoffs of the stage game under outsourcing (interpreted as market outcome), also satisfies Baker, Gibbons and Murphy's necessary and sufficient incentive constraints if players punish optimally and are sufficiently patient.*

Proof. Baker, Gibbons and Murphy show that the relational contract, that replicates the payoffs of the "market" or the stage game payoffs under outsourcing ω^O are given by $s = 0, b_{jk} = -\frac{1}{2}\left(Q_j + P_k\right)$. This implies $\max_{j,k=H,L} b_{jk} - \min_{j,k=H,L} b_{jk} = \frac{1}{2}\left(Q_H + P_H\right) - \frac{1}{2}\left(Q_L + P_L\right)$. To choose δ sufficiently high let

$$\begin{aligned} \delta &\geq \frac{\frac{1}{2}\left(Q_H + P_H\right) - \frac{1}{2}\left(Q_L + P_L\right)}{\frac{1}{2}\left(Q_H + P_H\right) - \frac{1}{2}\left(Q_L + P_L\right) + S^* - Q_L} \Leftrightarrow \\ \max_{j,k=H,L} b_{jk} - \min_{j,k=H,L} b_{jk} &\leq \frac{\delta}{1-\delta}\left(S^* - Q_L\right) \end{aligned}$$

which by the previous proof of proposition 4 corresponds to Baker, Gibbons and Murphy's necessary and sufficient incentive constraint for integration ω^I with optimal punishment. Since for this δ the neccessary and sufficient condition is satisfied under these assumptions this contradicts Baker, Gibbons and Murphy's claim that it is "impossible to bring the market inside the firm". ■

[24] The following inequalities go back to McLeod and Malcomson (1989) and correspond to (10) and (16) in the formulation of Baker, Gibbons and Murphy (2002) who only consider two ownership structures. See also there for a more detailed explanation and interpretation how to derive them in this context..

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