1 Introduction

Multiple contracts (lots) for the supply of similar goods or services are often procured sequentially at short distance one from the other. Sometimes this is done to simplify the awarding procedure since awarding many lots simultaneously is considered too complex and costly for small size sourcing by most procurement managers; other times to reduce the risk of mistakes and legal action, particularly in the case of public procurement. These sequences are also often regularly repeated in time, as is typical for most forms of procurement. Contracts for the supply of different but related goods - say, printers, laptops, desktops, monitors, servers - are often auctioned off sequentially. For multi-product suppliers active on several of these goods such contracts also represent sequential multi-lot procurement auctions regularly repeated in time.[1]

This paper discusses within a very simple model old and new reasons why recurrent sequential procurement of multiple lots may facilitate supplier collusion; then it identifies simple applicable policies to contain this problem based on (natural or induced) asymmetries among lots and on their place in the procurement sequence.[2]

Let us start recalling that preventing supplier collusion is a major problem for most types of procurement, even though some procurement managers appear still to be not aware of it.[3] According to Graham and Marshall (1987): "So prevalent are rings, in fact, that a retired auctioneer once noted that in 40 years of auctioneering, he had yet to attend an auction at which a ring was not present" (p. 1221). Procurement auctions are even more subject to the problem than other types of auctions, because suppliers typically interact repeatedly for a long time, know each other well and can coordinate their offers thanks to the transparency of (private and public) procurement processes, at

[1]Sequential *sale* auctions of multiple objects or lots are frequent in the electricity, timber, tobacco and oil lease markets, where they are also often repeated in time. Recurrent sequences of auctions, though common, appear not to have been studied in literature so far, and seem to be particularly subject to bidder collusion. In the US tobacco market, for example, sellers recently moved from a sequential open format to a sequential sealed bid first price format because bidders were perceived to collude. Government securities and mineral rights, on the other hand, are also sold repeatedly, but the multiple lots are generally sold in simultaneous auctions. We are grateful to Peter Cramton for suggesting all these examples.

[2]In the remainder of the paper we will use the words 'bidder' and 'supplier' as well as 'lot' and 'contract' as synonyms. Moreover, although our results apply also to repeated sequences of sale auctions, we will maintain focus on procurement as the interaction between suppliers is often so frequent and stable in time that preventing collusion becomes the single main problem for the procurer.

[3]See Albano et al. (2006) for an extensive discussion.

least in comparison with oligopolistic retail sales.[4]

A first familiar reason why sequential procurement may greatly facilitate supplier collusion is that it allows ring members to identify defections from collusive strategies and react to them within a sequence, reducing gains from defecting. The idea that 'fragmentation' facilitates cooperation for the good or for the bad was already stressed by Schelling (1960), formalized by Admati and Perry (1991) and Neher (1999), for joint projects and stage financing respectively, and developed in a procurement auctions context by Snyder (1996) (see also the informal discussion in Klemperer 2002).

A second, less familiar reason we highlight here is linked to possible asymmetries between cartel members. The viability of a collusive ring, as that of other types of cartels, is typically limited by "mavericks", that is by the bidder(s)/supplier(s) with the highest expected gains from undercutting the ring (e.g. Baker, 2002). When asymmetric members of a bidding ring share available lots and compensating transfers are risky, the ring can facilitate collusion by allocating to the most aggressive bidder the last lot in the sequence. This reduces the maverick's incentive to defect and stabilizes the ring.

These two pro-collusive effects of sequential auctions may interact with the asymmetry in the value of the lots and with their place in the sequence. After characterizing the role of lot asymmetry, we look for policies to fight bidder collusion. We show that if turning to a simultaneous setting is too costly, a simple and effective strategy against collusion is a 'large lot last' policy, that is, tendering the most valuable lot at the end of each sequence, so that the largest deviation cannot be punished before a new sequence of procurement lots starts, which may happen much further into the future.[5] Since the value of each of the lots procured and their place in the sequence are typically decisions in the hands of the procurer, implementing such a policy appears rather easy.

Our paper focuses on collusion in procurement auctions and on policies to prevent it, a literature partly surveyed in Klemperer (2002). In particular, our work is most closely related to papers like Wilson (1979), Robinson (1985),

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[4]Indeed, Porter and Zona (1993, 1999) detected bid rigging practices in a number of procurement auctions. Pesendorfer (2000) analyzes in depth the different bidding strategies of two long-lived school milk bidding rings. The phenomenon is so pervasive that it has often dominated antitrust enforcement. A General Accounting Office Report in 1990 stressed that between 1982 and 1988 over half of the criminal restraint of trade cases brought by the U.S. DoJ were linked to procurement auctions (see http://archive.gao.gov/d22t8/142779.pdf).

[5]We are then assuming that the procurer can only choose the order of the sequence. This implies that the procurer cannot decide to merge the two lots into a single contract. We thank F. Decarolis for pointing this out.

Graham and Marshall (1997), Snyder (1996) and at least in part Cramton and Schwartz (2000), as they also identify mechanical pro-collusive effects of aspects of competitive procedures that need not depend on the subtleties of the information structure, or on bidder communication or signalling.[6] Our results square well with the recent experimental evidence on the high susceptibility of sequential auctions to collusive rings in Dulatre and Sherstyuk (2008). The paper also contributes to the literature on simultaneous vs sequential format in multi-item auctions started by Weber (1983). In a recent survey of this literature, Feng and Chatterjee (2007) emphasize that the two auction formats have been studied mainly with respect to the quantity of information made available to bidders (when values are correlated) and to the mechanism designer's degree of commitment (when bidders have a single-unit demand). Here we show that the choice between sequential and simultaneous auctions/procurement should also depend on the risk of bid rigging, particularly when the sequences are frequently and regularly repeated.

Section 2 sets up the simplest possible model we could conceive and derives benchmark results; Section 3 introduces lot asymmetry and derives the collusion-preventing policy; Section 4 briefly concludes. Proofs are in an Appendix.

2 Set up and benchmarks

We adopt the simplest reduced form model we could conceive, as the forces we identify are intuitive and mechanical and therefore likely not to depend much on the specific formulation of the market game. In a more complex environment with imperfect information those forces should remain active and interact with other relevant forces, which may or may not outweigh them.

There are two 'relevant' long-lived potential suppliers, 1 and 2, among which the relevant characteristics (production costs for performing a procurement contract, discount factors, etc.) are common knowledge.[7] The simplifying assumption of complete information between bidders allows us to better focus on the effects of sequentiality and asymmetry and is not too unrealistic for many procurement situations, particularly in mature markets where suppliers are long-term competitors with a lot of personnel exchanges/turnover

[6]Models focussing on asymmetric information across bidders are much more complex. See for example, Hopenhayn and Skrzypacz (2004), Blume and Heidhues (2004) and Marshall and Marx (2007), among many others.

[7]The two firms could alternatively be considered as two "dominant firms" in presence of a fringe that has little chances to win; see Albano and Spagnolo (2005).

and knowledge of the market.

Two procurement contracts (say, geographical lots), named A and B, are awarded through competitive procurement processes, say lowest-price sealed-bid auctions, in every period t, $t = 1, 2...$, either sequentially or simultaneously within a negligible time frame. Suppliers discount time according to discount factors $0 < \delta_1, \delta_2 < 1$. We assume for simplicity and without loss of generality no discounting between the first and the second lot of a given sequence when procurement is sequential.

2.1 The fully symmetric benchmark case

Suppose temporarily that suppliers have the same discount factor and bear the same production cost for undertaking two equivalent procurement contracts, i.e. $\delta_1 = \delta_2 = \delta$, $c_1 = c_2 = c > 0$ and $A = B$. Assume also that the procurer commits to the same publicly announced reserve price, r, for each contract, with $r > c$. We find convenient to define $\overline{v} \equiv r - c$ as the upper bound to suppliers' profit from each procurement contract. The two contracts are repeatedly awarded through either a simultaneous or a sequential tendering format. In the latter case, all information is made public at the end of each round.

Since suppliers are identical and there is an upper bound on profits, the short-run competitive equilibrium in a one-shot competitive tendering is the Bertrand solution, delivering zero profits to each supplier (one can define these 0 extra profits, 0 being a normalization for the profits that can be earned from other activities). Suppose, further, that when suppliers decide to collude they support the ring by the threat of reverting to competitive behavior forever in case a deviation is observed. This punishment is optimal in this section's symmetric environment as it minimaxes the deviator, exactly as in a symmetric Bertrand supergame.

2.1.1 Split-award collusion

Suppose suppliers agree to collude by splitting each period, today and in the future, the two lots at the reserve price r. Sustaining split-award collusion in the future delivers $\overline{v}$ as net collusive profit to each supplier in every period, so that the discounted expected stream of collusive profits is $V^C = \frac{\overline{v}}{1-\delta}$, independent of whether a simultaneous or sequential procurement auction is used. As mentioned earlier, gains from defection (V^D) typically depend on whether the procurement auction used to allocate lots is sequential or simultaneous, because with sequential auctions opponents observe and can react earlier to the defection. However, in a symmetric environment this is only true if there are more than two lots to allocate. In fact, we have the following.

Lemma 1. *In a fully symmetric environment, with two lots awarded repeatedly and two bidders, the condition at which a stationary split-award collusive agreement is sustainable in equilibrium does not depend on whether the stage-game procurement auction is simultaneous or sequential.*

If a simultaneous format is adopted, each firm can deviate from the split award collusive agreement by marginally undercutting its competitor, thus getting one more lot at a price slightly below the agreed cartel price. So collusion is sustainable in equilibrium if

$$\begin{aligned} V^C &= \frac{\overline{v}}{1-\delta} \geq 2\overline{v} = V^D \Leftrightarrow \\ \delta &\geq \delta^{Split} = 1/2. \end{aligned} \tag{1}$$

Suppose now that the format is sequential, and that the cartel agreement allocates the first lot in the sequence to firm 1 and the second to firm 2. Firm 2 cannot gain a higher short-run profit than the collusive one by deviating - that is, by undercutting its competitor on the first lot - since it would be immediately detected and punished within the same period by firm 1. Thus firm 2's incentive compatibility constraint is trivially satisfied. Firm 1, however, faces the same constraint as under a simultaneous format, thus yielding the irrelevance result in Lemma 1.

2.1.2 Bid rotation

What if suppliers agree instead on a bid rotation scheme, whereby they take turns in getting both contracts A and B simultaneously at the reserve price? Consider the incentives of any of the two suppliers to defect in a period when, according to the bid rotation scheme, the other supplier should win both objects. If the former does not defect, it earns zero profits today but it expects discounted future profits $V^C = \delta \frac{2\overline{v}}{1-\delta^2}$. If it defects, in the future it will earn zero, but short run gains from defection *do* depend on which type of competitive format is adopted.

If the format is simultaneous a defecting supplier can earn $2\overline{v}$; if it is sequential, instead, it can only earn $\overline{v}$, as the opponent can observe the defection and react competing immediately on the second contract of the same sequence. Alternatively, and analogously, the defecting supplier might defect only on the second contract of the same sequence earning the same profit $\overline{v}$ and triggering Bertrand competition from the next stage afterwards.

Hence, with a sequential format the incentive constraint for a bid-rotation scheme being sustainable in equilibrium is

$$\begin{aligned} V^C &= \delta\frac{2\overline{v}}{1-\delta^2} \geq \overline{v} = V^D \Leftrightarrow \delta^2 + 2\delta - 1 \geq 0 \Leftrightarrow \\ \delta &\geq \delta^{RotSq} = \sqrt{2} - 1 < 1/2 = \delta^{Split}, \end{aligned}$$

and for the simultaneous auction is

$$\begin{aligned} V^C &= \delta\frac{2\overline{v}}{1-\delta^2} \geq 2\overline{v} = V^D \Leftrightarrow \delta^2 + \delta - 1 \geq 0 \Leftrightarrow \\ \delta &\geq \delta^{RotSm} = \frac{\sqrt{5}-1}{2} > \delta^{Split} > \delta^{RotSq}. \end{aligned}$$

This reasoning leads to the following:

Proposition 1. *In a fully symmetric environment, with two identical lots awarded repeatedly and two identical suppliers, the condition at which a bid-rotation collusive agreement is sustainable in equilibrium is more stringent if the stage-game procurement format is simultaneous than if it is sequential* ($\delta^{RotSm} > \delta^{RotSq}$)*. Moreover, the condition at which a bid-rotation equilibrium is sustainable is more stringent than the one at which a correspondent split award agreement is sustainable if a simultaneous procurement auction is used, whereas the opposite is true if a sequential one is used* ($\delta^{RotSm} > \delta^{Split} > \delta^{RotSq}$)*.*

In other words, a bid-rotation scheme together with a sequential auction re-activate Schelling's "fragmentation" effect even in a 2-lot-2-supplier environment, so that a bid rotation scheme becomes easier to sustain than a split the contracts scheme. The converse is true if a simultaneous auction is used.

Remark: *The results suggest that, if colluding bidders can choose among splitting the contracts and bid rotation and the discount factor is binding, then they will choose to split the contracts if the procurement auction is simultaneous, and bid rotation when the procurement auction is sequential.*

2.2 Asymmetric bidders and the 'Maverick' effect

The assumption of perfect symmetry between suppliers is admittedly restrictive. In this subsection we assume that the two bidders are asymmetric in terms of their incentives to collude, and that with a symmetric allocation of the collusive outcome one of the bidders' incentive compatibility constraints is violated and the other is slack. Suppliers might differ in capacities, hence ability to gain from deviations and punish, as in Comte *et al.* (2002); or, they can differ in costs, as in Miklos-Thal (2010); or else they can differ in their long-run fitness to survive in the market or access to financial resources, hence in discount factors. Whatever the source of the asymmetry, one of the two suppliers plays the role of the "maverick", as the latter's stronger incentive to defect limits the ability of the two suppliers to collude (see Baker 2002).

Suppose, further, that suppliers face transaction costs in redistributing collusive gains across the two bidders, through transfers or randomization schemes, to pool the two incentive constraints (for example because of the complexity of the redistribution schemes, or of the higher risk of being detected by the competition authority when operating such transfers). Then even at the optimal collusive scheme the incentive compatibility constraints of the two asymmetric bidders will not be identical, and one of the two suppliers may continue to play the role of a maverick that constrains their joint ability to sustain collusion.

We show here that in the split-award collusive agreement a sequential tendering format allows suppliers to eliminate the maverick effect by allocating to the maverick the last contract awarded within the same sequence. That is, if a sequential format is used instead of a simultaneous one, bidders can strictly improve on the optimal split-award collusive scheme by allocating the second lot in the sequence to the maverick and the first one to the supplier with a slack incentive constraint. Such an allocation scheme, by softening the maverick's incentive constraint, makes collusion easier to sustain. In fact, it turns out that this effect may induce suppliers to prefer a split-award collusive scheme to a bid rotation one. The result is in contrast with what found under full symmetry whereby the bid rotation is more effective than split-award (since, together with a sequential format, it revives Schelling's "fragmentation" effect).

Proposition 2. *When transfers are costly and bidders are sufficiently asymmetric (e.g. in discount factors) choosing a split-award arrangement and allocating the last lot to the 'maverick' facilitates collusion if the procurement auction is sequential.*

The proof is in the appendix and applies to other form of asymmetries. To illustrate the logic of the result, let us assume everything is symmetric but $\delta_1 < 1/2 < \delta_2$. If a simultaneous format is adopted each of the two firm $i's$ incentive compatibility constraint writes

$$\frac{\overline{v}}{1-\delta_i} \geq 2\overline{v},$$

for $i = 1, 2$. Collusion is therefore sustainable if and only if

$$\delta^{MavSm} \equiv \min\{\delta_1, \delta_2\} = \delta_1 \geq 1/2,$$

which since $\delta_1 < 1/2$ is not satisfied for the maverick. Under a sequential format however, if the cartel allocates the 'maverick' (firm 1) to the second object of the sequence, the maverick could only 'steal' the first lot, in which case it would be detected and would lose these gains because of the competition it triggered on the second lot. Hence the maverick's incentive constraint is never binding, the only relevant incentive constraint becomes firm 2's one

$$\delta^{MavSq} \equiv \delta_2 \geq 1/2,$$

and collusion becomes sustainable by the assumption that $1/2 < \delta_2$.

The simple policy conclusion we can draw from this is that a buyer should in general try *not* to use a sequential auction, and prefer instead a simultaneous one, when i) the risk of collusion among bidders is not negligeable; ii) the auctioned contracts are similar in value, but iii) bidders are so asymmetric in dimensions that it may affect their willingness to stick to a collusive agreement.

3 Lots asymmetry and the 'Large Lot Last' policy

Suppose now for simplicity that bidders are symmetric, as assumed in Section 2, but that lots are - or can be designed - to be heterogenous in value. That is, assume (w.l.o.g.) that $\overline{v}^A > \overline{v}^B$ and $\overline{v}^A + \overline{v}^B = 2\overline{v}$. Again, an optimal collusive scheme would aim at designing and implementing transfers between suppliers in order to re-allocate collusive gains symmetrically. However, as already discussed in the previous section, direct monetary transfers between competitors are complex to manage and increase the likelihood of a conviction by the competition authority. Equilibrating transfers between cartel members, however, could be operated intertemporally, either through a full bid-rotation scheme in which suppliers take turns in winning both lots at a collusive price; or by a partial rotation split-award collusive scheme in which suppliers split the lots in each period but take turns in receiving the more valuable one. We show here that when a sequential auction is used, it is more difficult for bidders to collude if the more valuable lot is always auctioned at the of *end* of the sequence.

3.1 Simultaneous procurement

Consider first the case in which a simultaneous procurement auction is used. Suppliers can still choose between a full bid-rotation scheme, in which suppliers take turns winning both contracts, and a split-award scheme where suppliers alternate in getting the larger lot A. It is easy to verify that with a *full rotation scheme* suppliers' incentive constraints coincide with those in the case of symmetric lots analyzed in Section 2: short run gains from defection are the sum of lots value, which is still $2\overline{v}$; the punishment phase is exactly like in Section 2. Hence, full collusion with full bid rotation is sustainable if and only if

$$\begin{aligned} 2\overline{v} &\leq \delta\frac{2\overline{v}}{1-\delta^2} \Leftrightarrow \\ \delta &\geq \delta^{RotSm} = \frac{\sqrt{5}-1}{2}. \end{aligned}$$

Consider now a partial rotation *split-award collusive* scheme. The more profitable defection consists in any supplier 'stealing' the more valuable contract in a period in which the agreement allocates it the less valuable one. The expected payoffs from such a defection would be $\overline{v}^A + \overline{v}^B$, as the payoffs in the following punishment phase are zero. Expected discounted profits from sticking to the collusive strategy, including that period's payoff, are instead $\overline{v}^B + \delta\frac{\overline{v}^A}{1-\delta^2} + \delta^2\frac{\overline{v}^B}{1-\delta^2}$. Hence, a partial rotation split-award scheme is sustainable via a simultaneous procurement auction if and only if

$$\begin{aligned} \overline{v}^A + \overline{v}^B &\leq \overline{v}^B + \delta\frac{\overline{v}^A}{1-\delta^2} + \delta^2\frac{\overline{v}^B}{1-\delta^2} \Leftrightarrow \qquad (2) \\ \delta &\geq \delta^{AsSplSm} = \frac{-\overline{v}^A + \sqrt{\left(\overline{v}^A\right)^2 + 4(\overline{v}^A + \overline{v}^B)\overline{v}^A}}{2(\overline{v}^A + \overline{v}^B)} < \delta^{RotSm}. \end{aligned}$$

Remark. *It is worth comparing how incentives to collude via a split-award scheme are modified when lots are asymmetric. Notice first that condition (2) can be rewritten as follows*

$$\frac{2\overline{v} - (1-\delta)v^A}{(1-\delta)(1+\delta)} \geq v^A + v^B = 2\overline{v}. \qquad (3)$$

When $v^A = v^B = \overline{v}$*, condition (3) trivially coincides with (1). When* $v^A > v^B$*, the higher the asymmetry between the two lots the higher the critical discount* *factor that is necessary to satisfy (3) (the left-hand side being increasing in δ). The simple intuition is that the higher the asymmetry the lower the cooperative payoff to the firm getting lot B in the first period. Thus that firm has to put a higher weight on future payoffs for finding it profitable to stick to the collusive agreement.*

3.2 Sequential procurement

Consider now the case in which a sequential procurement auction is used. As usual, suppliers can choose between a full bid rotation scheme, in which they take turns in winning both auctioned contracts, and a split-award scheme whereby they alternate in getting the larger contract A.

With a *full bid rotation* scheme suppliers' incentive constraints are slightly different than in the case of identical contracts analyzed in Section 2, as gains from defection are modified due to the asymmetry between lots. Consider the incentives of supplier $i \in \{1, 2\}$ to defect in a period when, according to the bid rotation scheme, supplier $j = 3-i$ should win both objects. If the supplier does not defect, it earns zero profits today but it expects discounted future profits $\delta\frac{\overline{v}^A+\overline{v}^B}{1-\delta^2} = \delta\frac{2\overline{v}}{1-\delta^2}$. If it defects, in the future it will earn zero, but short run gains from defection are now $\overline{v}^A > \overline{v}$. Regardless of whether lot A is auctioned first or second, the most profitable deviation consists in undercutting on that lot, after which the punishment starts, as the opponent can observe the defection and react competing already on the second lot of that sequence in case lot A was first. Hence, with a sequential auction the incentive constraint for a bid-rotation scheme being sustainable in equilibrium is

$$\begin{aligned} \overline{v}^A &\leq \delta\frac{\overline{v}^A+\overline{v}^B}{1-\delta^2} \Leftrightarrow \delta^2\overline{v}^A + \delta(\overline{v}^A+\overline{v}^B) - \overline{v}^A \geq 0 \Leftrightarrow \qquad (4) \\ \delta &\geq \delta^{AsRotSq} = \frac{-(\overline{v}^A+\overline{v}^B) + \sqrt{(\overline{v}^A+\overline{v}^B)^2 + 4\left(\overline{v}^A\right)^2}}{2\overline{v}^A} < \delta^{RotSm}. \end{aligned}$$

Consider now the crucial case of a partial rotation *split-award* scheme with the sequential format. The most profitable defection from the collusive scheme depends on whether the larger lot is auctioned before or after the smaller one. If the more valuable lot (A) is auctioned last, the most profitable defection is the one of the bidder who should get lot B: in fact, that supplier gets its lot first, and then steals the larger lot A undercutting its collusive partner. Collusion is then sustainable if and only if

$$\begin{aligned} \overline{v}^B + \overline{v}^A &\leq \overline{v}^B + \delta \frac{\overline{v}^A}{1-\delta^2} + \delta^2 \frac{\overline{v}^B}{1-\delta^2} \Leftrightarrow \quad (5) \\ \delta &\geq \delta^* = \frac{-\overline{v}^A + \sqrt{\left(\overline{v}^A\right)^2 + 4(\overline{v}^A + \overline{v}^B)\overline{v}^A}}{2(\overline{v}^A + \overline{v}^B)} = \delta^{AsSplSm}. \end{aligned}$$

If instead the large lot is auctioned always at the beginning, then maximum gains from defections are limited to $\max\left\{\overline{v}^A - \overline{v}^B, \overline{v}^B\right\}$. If the asymmetry is very strong, so that $\overline{v}^A > 2\overline{v}^B$, then the most profitable short-run defection is that of the supplier that should have got lot B at the second round but instead decided to undercut at the first round, obtaining extra profits $\overline{v}^A - \overline{v}^B$. Its incentive compatibility condition is then

$$\begin{aligned} \overline{v}^A - \overline{v}^B &\leq \overline{v}^B + \delta \frac{\overline{v}^A}{1-\delta^2} + \delta^2 \frac{\overline{v}^B}{1-\delta^2} \Leftrightarrow \quad (6) \\ \delta &\geq \delta^{'} \equiv \frac{-\overline{v}^A + \sqrt{\left(\overline{v}^A\right)^2 + 4(\overline{v}^A - 2\overline{v}^B)\overline{v}^A}}{2(\overline{v}^A + \overline{v}^B)} < \delta^* = \delta^{AsSplSm}. \end{aligned}$$

If instead asymmetry is not too large, $\overline{v}^A < 2\overline{v}^B$, then the maximum short-run gains from defection are those of the supplier that according to the collusive agreement obtains A first, and then decides to undercut to obtain also B at the second stage, obtaining $\overline{v}^B$ as net gains from defection. In this second case collusion is sustainable if and only if

$$\begin{aligned} \overline{v}^A + \overline{v}^B &\leq \overline{v}^A + \delta \frac{\overline{v}^B}{1-\delta^2} + \delta^2 \frac{\overline{v}^A}{1-\delta^2} \Leftrightarrow \quad (7) \\ \delta &\geq \delta^{''} \equiv \frac{-\overline{v}^B + \sqrt{\overline{v}^{B2} + 4(\overline{v}^A + \overline{v}^B)\overline{v}^B}}{2(\overline{v}^A + \overline{v}^B)} < \delta^{'} \mid_{\overline{v}^A < 2\overline{v}^B} < \delta^*, \delta^{AsSplSm}. \end{aligned}$$

The above inequalities allow us to state our main result.

Proposition 3. **('Large Lot Last' policy)** *Suppose that bidders are symmetric, lots are (or can be made) asymmetric and that a sequential procurement auction must be used in each period. Then procuring the most valuable lot at the END of the sequence minimizes bidders' ability to collude by making it identical to when a simultaneous procurement auction is used* ($\delta^{RotSm} > \delta^* = \delta^{AsSplSm}$).

The policy continues working when bidders have asymmetric valuations but the asymmetry is small relative to that in lots value. To see this, one could define $\overline{v}_i^k$ as bidder i's maximum profit from competing for contract k. Proposition 3 tells us that when $\overline{v}_1^A = \overline{v}_2^A = \overline{v}^A > \overline{v}^B = \overline{v}_1^B = \overline{v}_2^B$ the "large lot last" policy in a sequential format makes collusion as sustainable as under a simultaneous format. However, by continuity, the result is robust under small perturbations such as $\overline{v}_1^A \approx \overline{v}_2^A$ and $\overline{v}_1^B \approx \overline{v}_2^B$, that is, so long as a slight asymmetry arises between bidders' valuations of the same contract. Sufficient asymmetry in lots value creates robustness of this policy relative to asymmetries in bidders' valuations, and can be achieved whenever the procurer can purposely design lots of different value.

4 Discussion and conclusions

Groups of multiple procurement contracts are often and repeatedly awarded over time by means of a sequential competitive auctions. We have shown that the buyer may try to hamper suppliers' attempts to collude by generating and exploiting heterogeneity in lots value.

When the number of contracts goes up, for a fixed number of bidders, collusion becomes easier under a sequential format. First, bidders are able to exploit a growing number of allocations of lots within the same sequence in order to better align incentive constraints, thus remedying the destabilizing effect of the "maverick" bidder. Second, the longer the sequence of contracts awarded at each date the less attractive profits from defection on a single contract. We see no reasons, however, why a larger number of lots compared to the number of bidders could undermine the anti-collusive effect of tempting members of a ring by procuring the most valuable lot at the end of each sequence.

Arguably, a further dimension of asymmetry may arise. When bidders have opposite ranking of the two contracts - e.g., $\overline{v}_1^A > \overline{v}_2^A$ and $\overline{v}_1^B < \overline{v}_2^B$ - incentives to cooperate and, symmetrically, temptations to defect are not aligned anymore. The characterization of bidder optimal strategies and effective anti-collusive policy becomes then much more cumbersome. However, situations where bidders' idiosyncratic valuations are highly non-monotone in the value of lots are uncommon in regular procurement. Also, in these situations valuations are more likely to be private information, making bidders' optimal strategies much more complex and optimal policies conditional on that information, and as such of little practical value since the procurer would not know how to apply them. Still, that case would have theoretical interest, so we regard it as a fruitful avenue for future work.

5 Appendix

Proof of Proposition 2. Suppose that transfer schemes are prohibitively costly, and that supplier 1 is the 'maverick', so that the latter's incentive constraint is not satisfied under any of the collusive schemes discussed in the previous section, that is, $\delta_1 < \delta^{RotSq}$. Suppose, instead, that supplier 2 has strong incentives to stick to collusion, that its incentive constraint is satisfied at all the collusive paths discussed in the previous section, that is, $\delta_2 > \delta^{Split} > \delta^{RotSq}$.

If a simultaneous format is used, the collusive agreement is not sustainable because supplier 1, the maverick, would defect ($\delta_1 < \delta^{Split} < \delta^{RotSm}$).

If a sequential format is used, and bidders choose a bid-rotation collusive scheme, the agreement is not sustainable because supplier 1 would defect from it ($\delta_1 < \delta^{RotSq}$).

If a sequential format were adopted, and bidders chose instead a split-award agreement, they could also agree that the first auctioned contract in each period to be assigned to supplier 2, and the second auctioned contract to supplier 1, the maverick. Consider the profitability of defections from this *maverick-last* agreement. Supplier 2 will not defect because we assumed $\delta_2 > \delta^{Split}$. Supplier 1, the maverick, would defect if it were to be assigned the first contract ($\delta_1 < \delta^{RotSq}$), but would gain nothing from defecting if it were allocated the second contract. The only feasible defection then is to steal supplier 2's contract. However, if supplier 1 does so supplier 2 would revert to Bertrand competition immediately afterwards, so that supplier 1's gains from defection are zero. Therefore it is strictly better also for the maverick not to defect. ■

Proof of Proposition 3. We know already that the second part of the statement holds since the relevant incentive compatibility constraints (5) and (2) coincide. We have then to prove that the critical discount factor for collusion to be sustainable is highest under the 'large lot last' policy, namely that $\delta^* > \delta', \delta'', \delta^{AsRotSq}$. Instead of comparing directly the expressions of each discount factor we will proceed by comparing the relevant incentive compatibility constraints.

i) $\delta^* > \delta^{AsRotSq}$. Consider inequalities (5) and (4). By adding $\frac{(1-\delta)\overline{v}^B}{1-\delta^2}$ to both sides of (4) one can easily see that the left-hand sides (i.e., the discounted payoffs from sticking to the collusive strategies) of both (5) and (4) coincide.

We are left then to show that the right-hand side of (5) is always higher than that of (4), that is,

$$\overline{v}^A + \overline{v}^B \geq \overline{v}^A + \frac{(1-\delta)\overline{v}^B}{1-\delta^2} = \overline{v}^A + \frac{1}{1+\delta}\overline{v}^B$$

which is always true since $\delta \in (0,1)$.

ii) $\delta^* > \delta'$. This is immediate since the left hand sides of both (5) and (6) coincide, whereas the right-hand side of (5) is strictly greater that of (6).

iii) $\delta^* > \delta''$. In this case the right-hand sides of both (5) and (7) coincide. We have to show that the left-hand side of (5) is strictly greater than that of (7), that is,

$$\begin{aligned} \overline{v}^A + \delta\frac{\overline{v}^B}{1-\delta^2} + \delta^2\frac{\overline{v}^A}{1-\delta^2} &> \overline{v}^B + \delta\frac{\overline{v}^A}{1-\delta^2} + \delta^2\frac{\overline{v}^B}{1-\delta^2} \Longleftrightarrow \\ \overline{v}^A + \delta\overline{v}^B &> \overline{v}^B + \delta\overline{v}^A \end{aligned}$$

which is always true since $\overline{v}^A > \overline{v}^B$ by assumption.■

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